Prices *vs.* Quantities: A Macroeconomic Analysis

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This draft has been superseded by our paper: "A Theory of Supply Function Choice and Aggregate Supply." This older version contains novel results on prices vs. quantities choice that may be useful to other authors.

Abstract

We extend the standard model of imperfectly competitive firms that make supply decisions under uncertainty to include a "choice of choices": firms decide whether to set a price in advance and supply at the eventual market-clearing quantity or set a quantity in advance and sell at the eventual market-clearing price. In general equilibrium, this "choice of choices" affects the propagation of macroeconomic shocks in stark qualitative ways. Under quantity-setting, money has no real effects and passes through fully into prices. Under price-setting, money has real effects and passes through imperfectly to prices. This asymmetry generates new monetary policy trade-offs: attempts to stabilize the economy can backfire by inducing a regime shift that renders monetary policy ineffective. Testing the theory, we find that contractionary monetary policy shocks are output-neutral and deflationary in quantity-setting regimes (e.g., much of the 1970s) and output-depressing and non-deflationary in price-setting regimes.

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1 Introduction

At the heart of modern macroeconomic models are firms that make supply decisions under uncertainty, due to inattention (Mankiw and Reis, 2002), contracting frictions (Taylor, 1980; Calvo, 1983), or organizational constraints (Klemperer and Meyer, 1989). A long tradition in macroeconomics restricts these firms' supply decisions to an important but narrow class: setting a price and committing to produce enough to meet *ex post* demand.[1]

In this paper, we enrich the baseline model to allow for firms to choose different supply schedules. In the spirit of the classic debate between Bertrand (1883) and Cournot (1838), we focus on the most extreme departure from price-setting, *quantity-setting*: that is, producing a fixed amount and selling it at the *ex post* market-clearing price. We allow optimizing firms to make a "choice of choices" between price-setting and quantity-setting to maximize profits. These approaches may be equally consistent with the aforementioned informational, contracting, or organizational constraints, but may strongly differ in their appeal to firms and in their downstream macroeconomic consequences.

We first study monopolistically competitive firms' "choice of choices" in partial equilibrium. We derive a formula for the relative benefits of price-setting and quantity-setting in terms of the elasticity of demand and four moments of firms' beliefs about demand, costs, and others' prices. The basic logic echoes Weitzman's (1974) classic "prices *vs.* quantities" analysis of regulation: agents choose the decision variable that best insulates their payoff from shocks on which they cannot condition. A key trade-off that emerges in our setting is that price-setting insulates the firm against demand shocks, while quantity-setting insulates the firm against shocks to aggregate prices.

We next embed firms' price-setting *vs.* quantity-setting choice in a monetary business-cycle model. Macroeconomic dynamics are drastically different when firms set quantities as opposed to prices. In the former case, money does not affect real output and passes through one-for-one into prices. In the latter case, money has positive effects on real output and passes through less than one-for-one into prices. We derive the equilibrium incentives for price- and quantity-setting and show that price- and quantity-setting equilibria can generate self-fulfilling macroeconomic volatility in demand and prices. Monetary policy rules intended to stabilize the economy can backfire by inducing a switch to a more volatile regime.

We finally provide evidence that the "choice of choices" is empirically relevant. First,

[1] Price-setting is assumed in classic models of inflation dynamics based on exogenous, infrequent adjustment (Taylor, 1980; Calvo, 1983), menu costs (Barro, 1972; Sheshinski and Weiss, 1977), and limited information (Mankiw and Reis, 2002; Woodford, 2003a; Maćkowiak and Wiederholt, 2009). This converse assumption of quantity-setting is applied in models of real fluctuations in which production responds to expected demand (e.g., Angeletos and La'O, 2010, 2013; Benhabib et al., 2015; Flynn and Sastry, 2022a,b)

estimating our formula for the comparative advantage of price-setting in the data, we find evidence for both price and quantity regimes in US data – the former through the 1960s, the Great Moderation, and the financial crisis, and the latter in the 1970s stagflation and the post-Covid inflation. Second, testing our prediction that monetary policy has state-dependent effects, we find that contractionary monetary shocks (Romer and Romer, 2004) control inflation but not output in quantity-setting regimes and output but not inflation in price-setting regimes. Taken together, these findings show that adding the prices *vs.* quantities choice improves the model's ability to match regime shifts in US macroeconomic dynamics and suggest that the policy trade-offs implied by our theory are realistic.

The Prices *vs.* Quantities Choice. We first study the choice of setting prices *vs.* quantities for a single firm. The firm operates a Cobb-Douglas production function and faces a constant price elasticity of demand. It maximizes dollar profits, deflated by the aggregate price, and multiplied by a real stochastic discount factor. It is uncertain about shocks to demand, input prices, productivity, the stochastic discount factor, and the aggregate price level, all of which are jointly lognormally distributed.

The firm chooses either its price or its quantity under this uncertainty. The assumption underlying this choice is that the firm's attentional, contracting, or organizational frictions preclude it from making decisions *after* the realizations of uncertainty, but do not constrain whether the firm's *ex ante* plan takes a price or quantity form. Moreover, as is conventional, we assume that the variable that the firm does *not* choose is determined by *ex post* market clearing. If a firm chooses a price, it produces the quantity on the demand curve; if a firm chooses a quantity, it sells at the price on the demand curve.

In this environment, we derive the following closed-form expression for the relative value of price-setting versus quantity setting, Δ, in terms of the price elasticity of demand $\eta > 1$ and four moments of beliefs:

$$\begin{aligned}\Delta = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\,\mathrm{Var}[\text{Demand}] - \eta\,\mathrm{Var}[\text{Price Level}] \\ &- 2\,\mathrm{Cov}[\text{Real Marg. Cost, Demand}] - 2\eta\,\mathrm{Cov}[\text{Real Marg. Cost, Price Level}]\Bigg)\end{aligned} \tag{1}$$

Firms prefer to set prices when $\Delta > 0$, and quantities otherwise.

Four terms determine incentives in Equation 1. First, uncertainty about demand favors price-setting. Intuitively, the *ex post* optimal relative price is a fixed markup over real marginal costs *regardless of demand*—in this way, price-setting is hedged against unknown demand. Second, uncertainty about the price level favors quantity-setting. A price-setting firm needs to know aggregate prices to scale its price, while a quantity-setting firm does not—in this way, quantity-setting is hedged against unknown aggregate prices. Third and fourth, a positive covariance of real marginal costs with demand or the price level favors quantity-setting. In either case, price-setters mistakenly produce more exactly when marginal costs are high, amplifying their *ex post* profit losses.

The prices *vs.* quantities choice is a special case of the general problem of choosing *supply functions*, as studied by Klemperer and Meyer (1989) in the case of oligopoly. In an extension, we study how firms optimally choose flexible supply schedules. The globally optimal supply schedule is log-linear and nests pure price- and quantity-setting in special cases that are consistent with the trade-offs described above. We proceed in the remainder of the analysis to focus on the prices *vs.* quantities choice, to simplify our description of equilibrium while keeping the essence of the economic mechanism.

Macroeconomic Model and Implications. To understand the equilibrium implications of the prices *vs.* quantities choice, we embed it in a monetary business-cycle model with incomplete information, following Woodford (2003a) and Hellwig and Venkateswaran (2009). In addition to exogenous microeconomic and macroeconomic uncertainty, the model generates endogenous macroeconomic uncertainty about firms' demand, aggregate prices, and real marginal costs. In particular, because of imperfect competition between firms, the model features aggregate demand externalities (Blanchard and Kiyotaki, 1987) whereby firms face greater demand when aggregate output is high. Moreover, because households demand money, both the level of the money supply and aggregate output jointly determine the aggregate price level. Finally, because of income effects in labor supply, real marginal costs are higher when aggregate output is higher.

We first study how aggregate outcomes evolve in *temporary equilibria* in which firms' price-setting and quantity-setting decisions are taken as given. We find that monetary shocks are neutral under quantity-setting and affect aggregate prices one-for-one. Intuitively, if firms set quantities, any increase in demand that a monetary expansion may induce can never be met by a commensurate increase in supply if firms imperfectly respond to the monetary expansion. Thus, the price-level must adjust one-for-one to clear the goods market and aggregate output does not change in equilibrium. By contrast, under price-setting, monetary shocks have effects on real output and affect aggregate prices less than one-for-one. The basic logic behind this result echoes Lucas (1972): because firms are uncertain of the money supply, they do not increase their prices one-for-one in response to monetary shocks, and so monetary shocks can have real effects on the economy.

Having studied how the economy evolves under price- and quantity-setting, we return to the *general-equilibrium* version of our original question: when do equilibrium regimes of price- and quantity-setting exist? We first show that "choices of choices" are strategic complements: when other firms choose prices, a given firm has stronger incentives to choose prices. The intuition is sharpest when the economy is driven solely by money shocks: the price-setting regime induces more output volatility and less price volatility, which further favors price-setting. Building on this observation, we characterize conditions under which each equilibrium exists and under which both equilibria exist as functions of model primitives. The latter allows the model to generate time-varying macroeconomic volatility as the economy switches between price-setting and quantity-setting regimes.

Finally, we study monetary policy transmission. We ask how the extent to which the central bank "leans into" or "leans against" productivity shocks affects macroeconomic volatility and the possibility of both price-setting and quantity-setting regimes. Monetary policy that leans against productivity shocks can stabilize real output in a price-setting regime. Moreover, while monetary policy cannot affect real outcomes under quantity-setting, it does affect the volatility of aggregate prices and therefore the relative likelihood that the economy switches to a quantity-setting regime. Thus, monetary stabilization policy can run the risk of destabilizing the economy by inducing a switch into a higher volatility, quantity-setting regime. Moreover, our theory implies that policymakers face a *state-dependent* "Phillips curve." In particular, there is a trade-off between reducing aggregate prices and keeping real output high if and only if the economy is in a price-setting regime.

Taking the Model to the Data. To assess whether these trade-offs are empirically relevant, we estimate a time series for the relative advantage of price-setting (Δ from Equation 1) in US data from 1960 to the present. We discipline parameters using a GARCH model for the stochastic volatility of macroeconomic aggregates, a calibrated demand elasticity based on the estimates of Broda and Weinstein (2006), and a calibrated ratio of microeconomic to macroeconomic demand uncertainty based on the estimates of Bloom et al. (2018).

Using these methods, we find that both price-setting and quantity-setting are optimal at different points of US macro history. Price-setting is optimal for most (87%) of the sample, when inflation is relatively tame (the 1960s or the Great Moderation) and/or when demand volatility spikes (the Great Recession or Covid-19 Lockdown). Quantity setting is optimal when inflation volatility is high relative to demand volatility, as we estimate for much of the 1970s and the post-Covid-Lockdown inflation. In summary, the data do not support the assumption of time-invariant price-setting or quantity-setting. They instead suggest *regime shifts* as incentives move over time.

We next test the model's key macroeconomic prediction: monetary expansions increase real output more in price-setting regimes and increase prices more in quantity-setting regimes (and *vice versa* for contractions). We do so by estimating impulse responses of output (real

GDP) and prices (GDP deflator) to Romer and Romer (2004) monetary policy shocks, interacted with an indicator for whether firms would set prices according to our calculation of price-setting's comparative advantage.

We find, consistent with the theory, that monetary expansions have a relatively more positive effect on real output and a relatively more negative effect on prices in price-setting regimes. Strikingly, we cannot reject the null hypothesis of zero effect of monetary shocks on real GDP in quantity-setting regimes, while we find strong evidence of a negative effect in price-setting regimes. These results verify the novel prediction for macroeconomic dynamics that the "prices *vs.* quantities" mechanism implies.

Related Literature. The closest theoretical analysis to our paper is Reis (2006), who introduces a "prices *vs.* quantities" choice for a rationally inattentive firm. Studying a firm that faces general demand and cost curves, Reis derives an approximate condition for whether a firm should plan in prices or quantities. When the price elasticity of demand is constant, demand shocks are Gaussian and multiplicative, and cost shocks are independent of demand shocks, Reis shows that price-setting is preferred to quantity-setting. On the basis of this analysis, Reis concludes that price-setting is the better choice for firms. Our analysis differs from, and builds on, Reis' analysis in three ways. First, our partial-equilibrium analysis holds without approximation. Second, we study a case with uncertainty about multiple, correlated shocks, which we show is mostly relevant in business-cycle models in which costs and demand are endogenously co-determined. Third, we characterize the prices *vs.* quantities choice in equilibrium and study its implications for macro dynamics and policy.

Our work also relates theoretically to studies by Klemperer and Meyer (1986, 1989), in which the authors study oligopoly games under uncertainty with, respectively, price *vs.* quantity choice and supply-function choice. These authors' work on supply-function equilibrium relates to prior work by Grossman (1981) and Hart (1985) motivating supply-function choice as an outcome of realistic contracting and applying it to oligopoly without uncertainty. Our analysis share similar abstract motivations, but differs in studying monopolistic competition instead of oligopoly and embedding the findings in a macroeconomic model.

Our work's macroeconomic predictions relate to the literature on how uncertainty matters for the business cycle and *vice versa*. Previous work emphasizes how macroeconomic uncertainty affects firms' *quantitative* decisions, such as how much to produce (see, *e.g.,* Basu and Bundick, 2017; Bloom et al., 2018). By contrast, our analysis studies how the nature and extent of uncertainty about various factors affect the *qualitative* aspects of firms' choices about what to choose. Moreover, our theory offers a novel mechanism for endogenous macroeconomic uncertainty through variations in how firms make decisions. In so doing, our work relates to the literature that asks if the economy has time-varying volatility because of either time-varying shock sizes or because of time-varying responsiveness (see, *e.g.,* Berger and Vavra, 2019). Our analysis, however, emphasizes that time-varying volatility may itself generate time-varying responsiveness by changing the qualitative nature of firms' choices.

Finally, our findings regarding monetary policy relate to the literature on the state-dependent effects of monetary policy. This literature provides mixed evidence for whether monetary policy is more powerful (Weise, 1999; Garcia and Schaller, 2002; Lo and Piger, 2005) or less powerful (Tenreyro and Thwaites, 2016) in recessions. Our analysis differs in two respects. First, following our theory, our conditioning variable is not current or recent GDP, but instead the *comparative advantage of price-setting* which depends on (multiple) dimensions of uncertainty.[2] Second, unlike all aforementioned studies save Weise (1999), we jointly test for asymmetries in the responses of both output and prices.

Outline. The rest of the paper proceeds as follows. In Section 2, we perform our partial-equilibrium analysis. In Section 3, we present a monetary business-cycle model. In Section 4, we derive our theoretical characterization of price-setting and quantity-setting equilibria. In Section 5, we study the transmission of monetary policy. In Section 6, we apply our model to estimate our formula for the comparative advantage of price-setting. In Section 7, we test the macroeconomic implications of the theory by studying state-dependent effects of monetary policy. Section 8 concludes.

2 The Firm's Problem of Choosing What to Choose

We first study the problem of a single firm that must choose what to choose in the presence of uncertainty about demand, costs, aggregate prices, and the stochastic discount factor. We assume that the firm faces a constant price elasticity of demand, constant physical returns to scale, and jointly normal productivity, demand, input price, risk pricing, and aggregate price shocks. We derive a formula for the advantage of price-setting relative to quantity-setting in units of log expected profits. The formula conveys that price-setting is relatively more advantageous when demand volatility is high, aggregate price volatility is low, and the covariances of marginal costs with demand and aggregate prices are lower.

2.1 Set-up

We study the problem of a single firm that must fix either a quantity $q \in \mathbb{R}_+$ or a price $p \in \mathbb{R}_+$ in advance of fully knowing its costs, demand, the price level (i.e., the prices of

[2] A related point applies to Castelnuovo and Pellegrino's (2018) study of how the effects of monetary policy depend on "aggregate uncertainty." In our model, different components of uncertainty tip toward price-setting or quantity-setting, and therefore have opposite predictions for the effects of monetary policy.

competitors), and the stochastic discount factor. Our background assumption is that the firm faces frictions which necessitate making decisions *ex ante* but do not, by themselves, restrict the firm's planning instrument. This may be natural if firms can only take decisions when new information is available (Mankiw and Reis, 2002); if firms have infrequent opportunities to adjust contracts (Taylor, 1980; Calvo, 1983); or if the practical difficulties of implementing decisions in organizations necessitates only infrequently introducing new rules (Klemperer and Meyer, 1989). In each of these stories, we argue it is plausible that a firm could implement a price plan or a quantity plan.

The firm faces a constant-price-elasticity-of-demand demand curve given by:

$$\frac{p}{P} = \left(\frac{q}{\Psi}\right)^{-\frac{1}{\eta}} \tag{2}$$

where the random variable $\Psi \in \mathbb{R}_{++}$ is a stochastic demand shifter, the random variable $P \in \mathbb{R}_{++}$ is the aggregate price level, and $\eta > 1$ is the price elasticity of demand. The firm purchases bundles of inputs $x \in \mathbb{R}_{+}^{I}$ at random prices $p_x \in \mathbb{R}_{++}^{I}$ to produce according to a constant-returns-to-scale, Cobb-Douglas production function:

$$q = \Theta \prod_{i=1}^{I} x_i^{\alpha_i} \tag{3}$$

where the random variable $\Theta \in \mathbb{R}_{++}$ corresponds to the firm's Hicks-neutral productivity, and $\alpha_i \in \mathbb{R}_{++}$ is the input share of good i (with the property that $\sum_{i=1}^{I} \alpha_i = 1$). Finally, the firm's profits are priced according to a real stochastic discount factor represented by the random variable $\Lambda \in \mathbb{R}_{++}$.

The firm believes that the collection of random variables comprising demand, aggregate prices, productivity, input costs, and the stochastic discount factor $(\Psi, P, \Theta, \Lambda, p_x)$ is log-normal with mean μ and variance-covariance matrix Σ. Given this uncertainty, the firm seeks to maximize:

$$\mathbb{E}\left[\Lambda \frac{1}{P}\left(pq - p_x x\right)\right] \tag{4}$$

which is simply the expected value of real profits under the given stochastic discount factor.

Optimal Price-Setting. We first consider the problem of price-setting. If the firm sets a price p, it sells the quantity that clears markets *ex post*, or lies on the demand curve: $q = \Psi\left(\frac{p}{P}\right)^{-\eta}$. That is, the firm has committed to meeting demand at this price.

We now derive the optimal price. The cost of producing q is given by:

$$c(q; p_x, \Theta) = \min_{x \in \mathbb{R}_+^I} \sum_{i=1}^{I} p_{xi} x_i \quad \text{s.t.} \quad q = \Theta \prod_{i=1}^{I} x_i^{\alpha_i} \tag{5}$$

Taking first-order conditions, we obtain that the real cost function is given by:

$$\frac{c(q; p_x, \Theta)}{P} = \mathcal{M}(P, \Theta, p_x) q \tag{6}$$

with real marginal cost:

$$\mathcal{M}(P, \Theta, p_x) = P^{-1} \Theta^{-1} \prod_{i=1}^{I} \left(\frac{p_{xi}}{\alpha_i} \right)^{\alpha_i} \tag{7}$$

Thus, the problem of setting the optimal price reduces to:

$$V^P = \max_{p \in \mathbb{R}_+} \mathbb{E} \left[\Lambda \left(\frac{p}{P} - \mathcal{M} \right) \Psi \left(\frac{p}{P} \right)^{-\eta} \right] \tag{8}$$

Taking first-order conditions, the optimal price is given by:

$$p^* = \frac{\eta}{\eta - 1} \frac{\mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi \right]}{\mathbb{E}\left[\Lambda P^{\eta-1} \Psi \right]} \tag{9}$$

where the numerator is the expected marginal benefit of charging higher prices in reducing costs and the denominator is the expected marginal cost of charging higher prices in increasing revenue. In the absence of uncertainty, this reduces to the statement that the optimal relative price is a constant markup of $\frac{\eta}{\eta-1}$ on real marginal costs. Substituting the optimal price into the firm's payoff function and rearranging, we obtain that:

$$V^P = \frac{1}{\eta - 1} \left(\frac{\eta}{\eta - 1} \right)^{-\eta} \mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi \right]^{1-\eta} \mathbb{E}\left[\Lambda P^{\eta-1} \Psi \right]^{\eta} \tag{10}$$

Optimal Quantity Setting. We now study quantity-setting. If the firm sets a quantity q, it sells at the price that clears markets *ex post*: $p = P \left(\frac{q}{\Psi} \right)^{-\frac{1}{\eta}}$. This is the natural analogue of the *ex post* market clearing assumed with price-setting. In practice, it may reflect firms' ability to deploy managerial resources toward running an auction (Walrasian or otherwise) after demand is realized, at the cost of having to specify production in advance.

Applying the earlier steps, the problem of setting the optimal quantity reduces to:

$$V^Q = \max_{q \in \mathbb{R}_+} \mathbb{E}\left[\Lambda\left(\left(\frac{q}{\Psi}\right)^{-\frac{1}{\eta}} - \mathcal{M}\right) q\right] \tag{11}$$

The optimal quantity is given by:

$$q^* = \left(\frac{\eta}{\eta - 1}\frac{\mathbb{E}\left[\Lambda\mathcal{M}\right]}{\mathbb{E}\left[\Lambda\Psi^{\frac{1}{\eta}}\right]}\right)^{-\eta} \tag{12}$$

where the numerator is the expected marginal cost of expanding production and the denominator is the expected marginal revenue from expanding production. In the absence of uncertainty, this is the quantity that the firm sells by setting its relative price equal to a constant markup on its real marginal cost. Substituting the optimal quantity into the firm's payoff, we obtain:

$$V^Q = \frac{1}{\eta - 1}\left(\frac{\eta}{\eta - 1}\right)^{-\eta} \mathbb{E}\left[\Lambda\mathcal{M}\right]^{1-\eta} \mathbb{E}\left[\Lambda\Psi^{\frac{1}{\eta}}\right]^{\eta} \tag{13}$$

2.2 Result: When to Set Prices *vs.* Quantities

A cursory inspection of the values of price-setting and quantity-setting (Equations 10 and 13) reveals that they are not generally equal. We now characterize the relationship between the two and study the conditions under which each is preferred. Define the log-difference between the values of price-setting and quantity-setting as:

$$\Delta = \log V^P - \log V^Q \tag{14}$$

We obtain that:

$$\Delta = \eta\Bigg(\log \mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right] - \log \mathbb{E}\left[\Lambda\Psi^{\frac{1}{\eta}}\right]\Bigg) - (\eta - 1)\Bigg(\mathbb{E}\left[\Lambda\mathcal{M}P^{\eta}\Psi\right] - \mathbb{E}\left[\Lambda\mathcal{M}\right]\Bigg) \tag{15}$$

where we call the first term the "revenue-hedging" benefit of prices over quantities and the second term the "cost-hedging" cost of prices over quantities.

Under our log-normality assumption on the distribution of $(\Psi, P, \Theta, \Lambda, p_x)$, we have that $(\Psi, P, \Lambda, \mathcal{M})$ is also log-normal. Thus, we can analytically evaluate these expectations and compute their differences. Performing these calculations, we obtain the following formula (that we claimed in Equation 1) for the proportional benefit of prices over quantities:

Proposition 1 (Prices *vs.* Quantities)**.** *The comparative advantage of prices over quantities is given by:*

$$\Delta = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_\Psi^2 - \eta\sigma_P^2 - 2\sigma_{\Psi,\mathcal{M}} - 2\eta\sigma_{P,\mathcal{M}}\right) \tag{16}$$

Proof. See Appendix A.1. □

This formula expresses the relative benefit of prices over quantities in terms of a single structural parameter, the price elasticity of demand, and the following four moments: the variance of demand shocks, the variance of the aggregate price, the covariance between demand shocks and real marginal costs, and the covariance between the aggregate price and real marginal costs. In the four relevant moments, price-setting is relatively better than quantity-setting when (i) the volatility of demand σ_Ψ^2 is high, (ii) the volatility of aggregate prices σ_P^2 is low, (iii) the covariance between demand and real marginal costs is low, and (iv) the covariance between aggregate prices and real marginal costs is low. In the absence of uncertainty, $\Delta = 0$ and the firm is indifferent between setting prices or quantities. Finally, the proof in Appendix A.1 reveals that the distribution of the stochastic discount factor drops out of the calculation; thus, Δ is in units of log expected real profits.

To understand the intuition for (i)-(iv), we go case by case. First, in the presence of demand shocks alone, setting relative prices equal to a constant markup on marginal costs coincides with the first-best. By contrast, fixing the quantity supplied induces losses. Thus, demand shocks favor price-setting. Second, in the face of aggregate price shocks, fixing an optimal quantity allows relative prices to adjust perfectly while fixing an optimal price leads the firm's price to diverge from the aggregate price and loses revenue. Thus, aggregate price shocks favor quantity-setting. Third and fourth, when demand and real marginal costs or aggregate prices and real marginal costs negatively covary, price-setting causes the firm to produce a large amount exactly when costs are low, favoring price-setting. The extent to which the firm values (i)-(iv) is mediated by the price elasticity of demand, as that determines how rapidly prices respond to underlying changes.

As important as what does appear is what does not appear. First, in light of constant physical returns-to-scale, no means of any variables appear. Second, no moments involving the stochastic discount factor appear. This is somewhat surprising as the proof of the result shows that both the revenue-hedging benefits and cost-hedging costs depend on the properties of the stochastic discount factor. However, the stochastic discount factor enters both of these terms symmetrically, and its properties are therefore immaterial for the comparison of price-setting and quantity-setting. Third, the variance of real marginal costs does not appear as both quantity and price-setting manage real marginal cost variation equally well under constant physical returns.

We finally observe that the advantage of price-setting in Equation 16 could be empirically estimated if we could measure the elasticity of demand and firms' uncertainty about demand, the price level, and marginal costs. This calculation relies on the model structure of our firm's problem, but not on the specific general equilibrium closure we will pursue in Sections 3, 4, and 5. We will do such a calculation in Section 6.

2.3 Extensions: Decreasing Returns to Scale, Adjustment Costs, and Flexible Supply Schedules

Decreasing Returns to Scale. Our analysis so far assumed that firms have constant physical returns-to-scale and take input prices as given. In Appendix C.1, we derive a version of Proposition 1 when firms have decreasing returns and have monopsony power in input markets. This formula has many additional terms relative to Proposition 1, including the fact that means matter because of scale effects. This notwithstanding, the same core trade-off that we have highlighted, between demand uncertainty favoring price-setting and aggregate price uncertainty favoring quantity-setting, continues to hold.

Adjustment Costs. Our model assumes that there are no *direct* costs to *ex post* variation in prices or quantities, holding fixed their effects on (discounted) profits. However, in practice, adjusting quantities *ex post* could be costly because it is difficult to deploy (or hoard) factors, and adjusting prices *ex post* could be costly because it confuses or upsets consumers. To capture these forces, we pursue an extension in Appendix C.2 which allows for adjustment costs proportional to the unexpected variance of log quantities and log prices and provide the analogue to Proposition 1. Intuitively, quantity variance penalties favor quantity-setting and price variance penalties favor price-setting. Nonetheless, the key considerations described in our main analysis survive. In this way, adjustment costs might tilt the balance toward either prices or quantities, but not upset the logic that prices become more favorable with high demand variance and low price-level variance.

3 A Monetary Macroeconomic Model

We now embed the problem of "choosing what to choose" in a monetary macroeconomic model. We intentionally use standard microfoundations (see *e.g.,* Woodford, 2003b; Hellwig and Venkateswaran, 2009; Drenik and Perez, 2020) and deviate only in allowing firms to choose whether to commit to price or quantity choice. In Section 4, we use this model to derive a fully micro-founded, general-equilibrium specialization of Proposition 1 and to study the equilibrium implications of price vs. quantity choice. In Section 5, we extend this model to study the effects of monetary stabilization policies.

3.1 Households

Time is discrete and infinite $t \in \mathbb{N}$. A representative household has expected discounted utility preferences with discount factor $\beta \in (0,1)$ and per-period utility defined over a continuum of consumption varieties indexed by $i \in [0,1]$, C_{it}, holdings of real money balances, $\frac{M_t}{P_t}$, and labor effort supplied to each firm, N_{it}:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \ln \frac{M_t}{P_t} - \int_{[0,1]} \phi_{it} N_{it} \, \mathrm{d}i \right) \right] \tag{17}$$

where $\gamma \geq 0$ indexes income effects in both money demand and labor supply and $\phi_{it} > 0$ is the marginal disutility of labor supplied to firm i at time t, which is IID and follows $\log \phi_{it} \sim N(\mu_\phi, \sigma_\phi^2)$. The consumption aggregate C_t is a constant-elasticity-of-substitution aggregate of the individual consumption varieties with elasticity of substitution given by $\eta > 1$:

$$C_t = \left(\int_{[0,1]} \vartheta_{it}^{\frac{1}{\eta}} c_{it}^{\frac{\eta-1}{\eta}} \, \mathrm{d}i \right)^{\frac{\eta}{\eta-1}} \tag{18}$$

where ϑ_{it} is an IID preference shock and follows $\log \vartheta_{it} \sim N(\mu_\vartheta, \sigma_\vartheta^2)$. We also define the corresponding ideal price index:

$$P_t = \left(\int_{[0,1]} \vartheta_{it} p_{it}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{19}$$

Households can save in either money or risk-free one-period bonds B_t (in zero net supply) that pay an interest rate of $(1+i_t)$. The household owns the firms in the economy, each of which has real profits of Π_{it}. Thus, the household faces the following budget constraint at each time t:

$$M_t + B_t + \int_{[0,1]} p_{it} C_{it} \, \mathrm{d}i = M_{t-1} + (1+i_{t-1}) B_{t-1} + \int_{[0,1]} w_{it} N_{it} \, \mathrm{d}i + \int_{[0,1]} \Pi_{it} \, \mathrm{d}i \tag{20}$$

where p_{it} is the price of variety of variety i and w_t is the nominal wage.

The aggregate money supply follows a random walk with drift μ_M and time-dependent volatility $\{\sigma_t^M\}_{t \in \mathbb{N}}$:

$$\log M_t = \log M_{t-1} + \mu_M + \sigma_t^M \varepsilon_t^M \tag{21}$$

where the money innovation is an IID random variable that follows $\varepsilon_t^M \sim N(0,1)$. So that interest rates remain strictly positive, we assume that $\frac{1}{2}\sigma_{M,t}^2 \leq \mu_M$ for all $t \in \mathbb{N}$.

3.2 Firms

The production side of the model follows closely the model from Section 2. Each consumption variety is produced by a separate monopolist, also indexed by $i \in [0,1]$. The firm can choose to set a quantity q_{it} or a price p_{it}. The firm hires labor L_{it} (the sole input in this model) at wage w_{it} to produce according to the constant returns-to-scale production technology:

$$q_{it} = z_{it} A_t L_{it} \tag{22}$$

where z_{it} is IID and follows $\log z_{it} \sim N(\mu_z, \sigma_z^2)$ while $\log A_t$ follows an AR(1) with time-varying volatility σ_t^A:

$$\log A_t = \rho \log A_{t-1} + \sigma_t^A \varepsilon_t^A \tag{23}$$

with the productivity innovations are IID and follow $\varepsilon_t^A \sim N(0,1)$. The firm's nominal profits are given by:

$$\Pi_{it} = p_{it} q_{it} - w_{it} L_{it} \tag{24}$$

Since firms are owned by the representative household, their objective is to maximize expectations of real profits, discounted by some stochastic discount factor, or $\frac{\Lambda_t}{P_t}\Pi_{it}$.

At the beginning of time period t, firms first observe A_{t-1} and M_{t-1}. Second, firms receive private signals about aggregate productivity s_{it}^A and the money supply s_{it}^M:

$$\begin{aligned} s_{it}^A &= \log A_t + \sigma_{A,s} \varepsilon_{it}^{s,A} \\ s_{it}^M &= \log M_t + \sigma_{M,s} \varepsilon_{it}^{s,M} \end{aligned} \tag{25}$$

where the signal noise is IID and follows $\varepsilon_{it}^{s,A}, \varepsilon_{it}^{s,M} \sim N(0,1)$. Third, firms choose whether to set a price or a quantity and what to set it at. Fourth, the money supply, idiosyncratic demand shocks, and both aggregate and idiosyncratic productivity are realized. Finally, the household makes its consumption and savings decisions and any prices that were not fixed adjust to clear the market.

3.3 Equilibrium

We define equilibrium in two steps. We first fix firms' "choice of choices" at each date t to define a rational expectations *temporary equilibrium*:

Definition 1 (Temporary Equilibrium)**.** *A temporary equilibrium is a partition of* $\mathbb{N}$ *into two sets* $\mathcal{T}^P$ *and* $\mathcal{T}^Q$ *and a collection of variables*

$$\left\{\{p_{it}, q_{it}, C_{it}, N_{it}, L_{it}, w_{it}, \phi_{it}, \vartheta_{it}, z_{it}, \Pi_{it}\}_{i\in[0,1]}, C_t, P_t, M_t, A_t, B_t, N_t, \Lambda_t, \sigma_t^A, \sigma_t^M\right\}_{t\in\mathbb{N}} \tag{26}$$

such that:

1. *In periods* $t \in \mathcal{T}^P$*, all firms choose their prices* p_{it} *to maximize expected real profits under the household's real stochastic discount factor.*
2. *In periods* $t \in \mathcal{T}^Q$*, all firms choose their quantities* q_{it} *to maximize expected real profits under the household's real stochastic discount factor.*
3. *In all periods, the household chooses consumption* C_{it}*, labor supply* N_{it}*, money holdings* M_t*, and bond holdings* B_t *to maximize their expected utility subject to their lifetime budget constraint, while* Λ_t *is the household's marginal utility of consumption.*
4. *In all periods, money supply* M_t *and productivity* A_t *and evolve exogenously via Equations 21 and 23.*
5. *In all periods, firms' and consumers' expectations are consistent with the equilibrium law of motion.*
6. *In all periods, the markets for the intermediate goods, final good, labor varieties, bonds, and money balances all clear.*

In a temporary equilibrium, firms set either prices or quantities, but the choice between the two is not necessarily optimal. We define an *equilibrium* as a temporary equilibrium in which the choice between price and quantity-setting is optimal at all times:

Definition 2 (Equilibrium)**.** *An equilibrium is a temporary equilibrium in which:*

1. *If* $t \in \mathcal{T}^P$*, all firms find price-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*
2. *If* $t \in \mathcal{T}^Q$*, all firms find quantity-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*

4 Choosing What to Choose in General-Equilibrium

We now study the equilibrium properties of the model. We begin by deriving the structure of firms' demand and costs in equilibrium. Using this, we characterize the aggregate behavior of consumption and prices under quantity-setting and price-setting temporary equilibrium, in which all firms always use the respective planning instrument. If all firms set prices, monetary shocks have effects on real output. By contrast, if all firms set quantities, money is *neutral* and has no effect on real output. We finally characterize when price-setting and quantity-setting equilibria obtain and derive comparative statics for their presence in terms of the extent and nature of aggregate volatility.

4.1 Demand and Costs in Equilibrium

We begin by deriving each firm's demand curve, the aggregate price level, its real marginal costs, and the stochastic discount factor that they face in equilibrium. From the intratemporal Euler equation for consumption demand *vs.* labor supply, the household equates the marginal benefit of supplying additional labor $w_{it} C_t^{-\gamma} P_t^{-1}$ with its marginal cost ϕ_{it}. Thus, we have that:

$$w_{it} = \phi_{it} P_t C_t^{\gamma} \tag{27}$$

From the intertemporal Euler equation between consumption and money today, we obtain that the cost of holding an additional dollar today $C_t^{-\gamma} P_t^{-1}$ equals the benefit of holding an additional dollar today M_t^{-1} plus the value of an additional dollar tomorrow $\beta \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right]$:

$$C_t^{-\gamma} \frac{1}{P_t} = \frac{1}{M_t} + \beta \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] \tag{28}$$

Further, from the intertemporal choice between bonds, we know that the cost of saving an additional dollar today equals the nominal interest rate $1+i_t$ times tha value of an additional dollar tomorrow:

$$C_t^{-\gamma} \frac{1}{P_t} = \beta (1+i_t) \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] \tag{29}$$

By combining Equations 28 and 29, we obtain that aggregate consumption demand follows:

$$C_t = \left(\frac{i_t}{1+i_t} \right)^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t} \right)^{\frac{1}{\gamma}} \tag{30}$$

which implies that aggregate consumption is increasing in real money balances, with elasticity given by $\frac{1}{\gamma}$. Intuitively, when consumption has greater curvature, income effects in money demand are larger and money demand is more responsive to changes in consumption. Thus implies that consumption responds less to real money balances when γ is large. The level of real money balances naturally depends on the opportunity cost of holding money i_t, and so money demand is lower when interest rates are high, all else equal.

Moreover, by substituting Equation 30 back into Equation 29, we obtain a recursion that interest rates must satisfy:

$$\frac{1+i_t}{i_t} = 1 + \beta \mathbb{E}_t \left[\frac{1+i_{t+1}}{i_{t+1}} \frac{M_t}{M_{t+1}} \right] \tag{31}$$

As money follows a random walk, solving this equation forward and employing the household's transversality condition, we obtain that:[3]

$$\frac{1+i_t}{i_t} = 1 + \beta \exp\{-\mu + \frac{1}{2}\sigma_{M,t}^2\} \sum_{i=1}^{\infty} \prod_{j=1}^{i} \beta \exp\{-\mu + \frac{1}{2}\sigma_{M,t+j}^2\} \tag{33}$$

which is deterministic, but depends on the full future path of monetary volatility.

Finally, from the optimal choice between varieties, we obtain that the firm's demand curve is given by:

$$\frac{p_{it}}{P_t} = \left(\frac{c_{it}}{\vartheta_{it} C_t} \right)^{-\frac{1}{\eta}} \tag{34}$$

which expresses the idea that the firm faces strong demand when aggregate consumption is high, its competitors prices are low, or its idiosyncratic demand is high. Moreover, η is the price elasticity of demand that the firm faces.

Summarizing the above, we have derived the following equilibrium mapping from endogenous objects to the objects that are relevant to the firm in partial-equilibrium.

Lemma 1 (Firm-Level Shocks in General Equilibrium)**.** *In any temporary equilibrium, demand shocks, aggregate price shocks, stochastic discount factor shocks, and marginal cost shocks follow:*

$$\Psi_{it} = \vartheta_{it} C_t\,, \quad P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t\,, \quad \Lambda_t = C_t^{-\gamma}\,, \quad \mathcal{M}_{it} = \frac{\phi_{it} C_t^{\gamma}}{z_{it} A_t} \tag{35}$$

Proof. See Appendix A.2. □

The first expression conveys that demand shocks have two components: an idiosyncratic shock deriving from consumer preferences and an aggregate shock corresponding to the aggregate demand externality (Blanchard and Kiyotaki, 1987). The second expression derives from households' demand for money balances, and conveys the fact that the price level must

[3] Observe also that in the case of time-invariant money volatility, interest rates follow the familiar equation:

$$1 + i^* = \beta^{-1} \exp\left\{ \mu_M - \frac{1}{2}\sigma_M^2 \right\} \tag{32}$$

increase in nominal money balances, increase in the nominal interest rate, and decrease in consumption to lie on this demand curve. The third expression derives from the representative consumer's CRRA preferences. The fourth expression derives from combining the labor supply curve with the assumption that firms' productivity has a microeconomic component z_{it} and a macroeconomic component A_t. Finally, note that the presence of common macroeconomic variables in these four expressions necessarily implies covariances between these objects.

An important implication of Lemma 1 is that, if C_t is log-normal in a temporary equilibrium, then so too is $(\Psi_{it}, P_t, \Lambda_t, \mathcal{M}_{it})$. This follows from the fact that all four expressions are log-linear and all other fundamentals $(M_t, \vartheta_{it}, \phi_{it}, z_{it}, A_t)$ are log-normal by assumption. Therefore, if we can find that C_t is log-normal in a temporary equilibrium, our Proposition 1 can be directly applied to calculate the relative benefits of quantity-setting and price-setting in *general-equilibrium*. We will call a temporary equilibrium in which $\log C_t$ is linear in $(\log A_t, \log M_t)$ a *log-linear temporary equilibrium*.

4.2 Outcomes under Price-Setting and Quantity-Setting

We next characterize aggregate outcomes in the economy taking as given that all firms set either prices or quantities. In particular, we establish that there are indeed temporary equilibria in which aggregate consumption is exactly log-linear in aggregate shocks.

Outcomes Under Price-Setting. Suppose that all firms set prices. We guess and verify that there exists a unique temporary equilibrium in which aggregate consumption is log-linear in aggregate shocks:

$$\log C_t = \chi^P_{0,t} + \chi^P_{A,t} \log A_t + \chi^P_{M,t} \log M_t \tag{36}$$

Combining our formula for the optimal price (Equation 9) with Lemma 1, the optimal price follows:

$$\log p_{it} = \log\left(\frac{\eta}{\eta-1}\right) + \log \mathbb{E}_{it}\left[\phi_{it}(z_{it}A_t)^{-1} P_t^{\eta} \vartheta_{it} C_t\right] - \log \mathbb{E}_{it}\left[C_t^{1-\gamma} P_t^{\eta-1} \vartheta_{it}\right] \tag{37}$$

Substituting our guess that C_t is log-linear in the aggregate shocks, we obtain that p_{it} is log-linear in the firm's signals about the aggregate shocks. We can then aggregate the prices that firms set by exploiting the formula for the aggregate price along with log-normality of the signals and the idiosyncratic demand shocks:

$$\log P_t = \frac{1}{1-\eta} \log \mathbb{E}_t\left[\exp\left\{\log \vartheta_{it} + (1-\eta)\log p_{it}\right\}\right] \tag{38}$$

Substituting this into the household's consumption demand (Equation 30) yields a formula for aggregate consumption. As this is indeed log-linear, we can solve for the unique coefficients $(\chi^P_{0,t}, \chi^P_{A,t}, \chi^P_{M,t})$ that verify the conjecture. To this end, define:

$$\kappa^A_t = \frac{1}{1+\left(\frac{\sigma_{A,s}}{\sigma^A_t}\right)^2}, \qquad \kappa^M_t = \frac{1}{1+\left(\frac{\sigma_{M,s}}{\sigma^M_t}\right)^2} \tag{39}$$

which is the posterior weight on the firms' signals of productivity and the aggregate money supply. Performing the above steps yields the dynamics of the economy when all firms choose to set prices.

Proposition 2 (Outcomes under Price-Setting)**.** *If all firms set prices, output in the unique log-linear temporary equilibrium follows:*

$$\log C_t = \chi^P_{0,t} + \frac{1}{\gamma}\kappa^A_t \log A_t + \frac{1}{\gamma}\left(1-\kappa^M_t\right)\log M_t \tag{40}$$

and the aggregate price in the unique log-linear temporary equilibrium is given by:

$$\log P_t = \tilde{\chi}^P_{0,t} - \kappa^A_t \log A_t + \kappa^M_t \log M_t \tag{41}$$

where $\chi^P_{0,t}$ and $\tilde{\chi}^P_{0,t}$ are constants that depend only on parameters and past shocks to the economy.

Proof. See Appendix A.3. □

This result establishes that when all firms set prices, so long as information about the money supply is not perfect, monetary shocks affect real output and consumption. The basic logic echoes Lucas (1972). When the money supply increases by one percent, the partial-equilibrium effect is that real money balances increase by one percent and so consumption increases by $\frac{1}{\gamma}$ percent. This causes an increase in real wages by $\gamma \times \frac{1}{\gamma} = 1$ percent. Moreover, as firms imperfectly observe the money supply, when it goes up by one percent, on average firms perceive that it has gone up by κ^M_t percent. Hence, on average, firms think that real marginal costs will go up by κ^M_t percent. As firms charge a constant markup on their marginal costs, firms increase prices by κ^M_t percent on average. This is the partial-equilibrium effect. There are two general equilibrium effects. First, this κ^M_t percent increase in prices reduces real money balances by κ^M_t percent, which reduces consumption by $\frac{1}{\gamma}\kappa^M_t$ percent, which decreases perceived real marginal costs and prices by ${\kappa^M_t}^2$. Second, as prices have gone up by κ^M_t percent, all firms adjust their prices up by ${\kappa^M_t}^2$. These two general equilibrium effects perfectly offset. Thus, the total effect is simply the partial-equilibrium effect and prices rise by κ_t^M percent. Given this imperfect pass-through to equilibrium prices, the equilibrium effect on real money balances of a one percent expansion in the money supply is a $1-\kappa_t^M$ percent increase. Hence, aggregate consumption increases by $\frac{1}{\gamma}(1-\kappa_t^M)$ percent and changes in the money supply have effects on real output.

To understand the pass-through of productivity shocks under price-setting, suppose that productivity increases by one percent. When productivity increases by one percent, as firms have imperfect information, they perceive that it has increased by κ_t^A percent on average. Under price-setting, as firms charge a constant markup on marginal costs, a one percent decrease in the marginal cost of production passes through into a one percent decrease in their price. So the direct effect of this change is that firms all reduce their prices by κ_t^A percent. This induces the following general-equilibrium effects. First, if aggregate prices fall by one percent, then a firm reduces its price by one percent to prevent a fall in demand. Second, if aggregate prices fall by one percent, real money balances increase by one percent, and so consumption goes up by $\frac{1}{\gamma}$ percent. This causes an increase in real wages by $1=\frac{1}{\gamma}\times\gamma$ percent, which increases real marginal costs by one percent and leads to a one percent increase in prices. These two general-equilibrium effects perfectly cancel, and so the general-equilibrium effect of productivity on prices is equal to the partial-equilibrium effect of productivity on prices. Thus, prices fall by κ_t^A percent when productivity increases by one percent. Moreover, this causes a κ_t^A percent increase in real money balances, and so consumption increases by $\frac{1}{\gamma}\kappa_t^A$ percent.

Outcomes Under Quantity-Setting. Next, we suppose that all firms set quantities. We again conjecture that aggregate consumption is log-linear in aggregate shocks:

$$\log C_t = \chi_{0,t}^Q + \chi_{A,t}^Q \log A_t + \chi_{M,t}^Q \log M_t \tag{42}$$

where, as we derived in Section 2 the optimal quantity set by firms is given by

$$\log q_{it} = -\eta \left[\log\left(\frac{\eta}{\eta-1}\right) + \log \mathbb{E}_{it}\left[\phi_{it}\left(z_{it}A_t\right)^{-1}\right] - \mathbb{E}_{it}\left[\vartheta_{it}^{\frac{1}{\eta}} C_t^{-\gamma+\frac{1}{\eta}}\right]\right] \tag{43}$$

Substituting our guess that C_t is log-linear in the aggregate shocks, we can obtain an log-linear expression for q_{it}, which we may substitute into the consumption index (Equation 18). Aggregating quantities in this way yields a log-linear expression for aggregate consumption, which we can use to solve for the unique coefficients $\left(\chi_{0,t}^Q, \chi_{A,t}^Q, \chi_{M,t}^Q\right)$. Performing the above the above steps yields the dynamics of the economy when all firms choose to set quantities.

Proposition 3 (Outcomes under Quantity-Setting)**.** *If all firms set quantities, output in* *the unique log-linear temporary equilibrium follows:*

$$\log C_t = \chi_{0,t}^Q + \frac{\eta \kappa_t^A}{1 - \kappa_t^A (1 - \eta\gamma)} \log A_t \tag{44}$$

and the aggregate price in the unique log-linear temporary equilibrium is given by:

$$\log P_t = \tilde{\chi}_{0,t}^Q - \frac{\eta\gamma \kappa_t^A}{1 - \kappa_t^A (1 - \eta\gamma)} \log A_t + \log M_t \tag{45}$$

where $\chi_{0,t}^Q$ and $\tilde{\chi}_{0,t}^Q$ are constants that depend only on parameters and past shocks to the economy.

Proof. See Appendix A.4. □

Money supply shocks are neutral in quantity-setting equilibria, in an important contrast to the price-setting analysis of Proposition 2. The reason is subtle. Suppose that the money supply goes up by one percent. Absent any adjustment in prices, consumer demand would go up by $\frac{1}{\gamma}$ percent. This has two effects on firms' quantity-setting choices. First, firms on average perceive that wages and real marginal costs increase by $\kappa_t^M = \gamma \times \frac{1}{\gamma}\kappa_t^M$ percent. Second, firms on average perceive that aggregate demand increases by $\frac{1}{\gamma} \times \frac{1}{\eta}\kappa_t^M$ percent. Thus, the quantity that the firm sets increases by $\frac{1}{\gamma}\kappa_t^M \eta \left(\frac{1}{\eta} - \gamma\right)$ percent. However, for the goods market to clear, we require that the change in demand equals the change in supply, which requires that $\frac{1}{\gamma} = \frac{1}{\gamma}\kappa_t^M \eta \left(\frac{1}{\eta} - \gamma\right)$. This is equivalent to requiring that $1 = \kappa_t^M(1 - \eta\gamma)$. However, as $\eta\gamma \geq 0$ and $\kappa_t^M < 1$, this is impossible. Intuitively, even in the absence of income effects in labor supply, as firms imperfectly respond to any increases in demand that a monetary expansion might induce, supply can never meet demand. Thus, prices must increase until any increase in demand is perfectly offset, which requires that real money balances remain unchanged. Hence, there are no real effects of changes in the money supply and full pass-through of changes in the money supply into prices.

To understand the pass-through of productivity shocks under quantity-setting, we describe the partial-equilibrium effects on production and their general-equilibrium amplification. Under quantity-setting, when aggregate productivity goes up, firms on average think that aggregate productivity has gone up by κ_t^A as they observe this increase imperfectly. In response to a one percent productivity increase, firms increase production by η percent. This itself increases aggregate demand by $\frac{1}{\eta}$ percent through aggregate demand externalities, which increases production by $\eta \times \frac{1}{\eta} = 1$ percent. However, it also increases wages by γ percent because of income effects, which causes firms to reduce production by $\eta\gamma$ percent. Thus, the direct effects on production are $\kappa_t^A \times \eta$ and the *first-round* general-equilibrium effects are $\kappa_t^A \times (1-\eta\gamma)$. Iterating this logic through higher-round general-equilibrium effects, we obtain that:[4]

$$\frac{\partial \log C_t}{\partial \log A_t} = \underbrace{\eta\kappa_t^A}_{\text{PE}} + \underbrace{\eta\kappa_t^A \sum_{k=1}^{\infty} \left[\kappa_t^A \times (1-\eta\gamma)\right]^k}_{\text{GE}} = \frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)} \tag{46}$$

Thus, the pass-through of productivity shocks is increasing in how well firms perceive them κ_t^A and the strength of aggregate demand externalities η, while it decreases in the extent of income effects in labor supply γ.

Comparing Outcomes under Price-Setting and Quantity-Setting. We now summarize the key differences in how macroeconomic variables respond to shocks under the two regimes. We first formalize the sharp difference between how the economy responds to monetary shocks across the price-setting and quantity-setting regimes:

Corollary 1 (Differential Responses to Monetary Shocks)**.** *In the unique log-linear temporary equilibria under price-setting and quanity-setting, the responses of real output and the aggregate price to money shocks satisfy:*

$$\frac{\partial \log C_t^P}{\partial \log M_t} > \frac{\partial \log C_t^Q}{\partial \log M_t} = 0 \qquad 1 = \frac{\partial \log P_t^Q}{\partial \log M_t} > \frac{\partial \log P_t^P}{\partial \log M_t} > 0 \tag{47}$$

For the reasons we have described, money shocks have a higher pass-through into real consumption and a lower pass-through into prices in a price-setting economy versus a quantity-setting economy. We will test this prediction in Section 7. We next summarize the differential response to productivity shocks:

Corollary 2 (Differential Responses to Productivity Shocks)**.** *In the unique log-linear temporary equilibria under price-setting and quantity-setting, the responses of real output and the aggregate price to productivity shocks satisfy, when $\eta\gamma < 1$:*

$$\frac{\partial \log C^P}{\partial \log A} > \frac{\partial \log C^Q}{\partial \log A} > 0 \qquad \frac{\partial \log P^P}{\partial \log A} < \frac{\partial \log P^Q}{\partial \log A} < 0 \tag{48}$$

with the reverse inequalities (but the same sign) when $\eta\gamma > 1$ and equality (but the same sign) when $\eta\gamma = 1$.

[4]This logic is informal and the proof of the result uses formal fixed-point techniques. In particular, observe that if $\kappa_t^A(1-\eta\gamma) < -1$, then the geometric sum does not converge. Yet there still exists the claimed temporary equilibrium in this case.

Whether responses to productivity shocks are larger under price-setting or quantity-setting depends on the relationship of $\eta\gamma$, which determines the size of the partial-equilibrium and general-equilibrium effects of productivity shocks on output. Intuitively, they are both equal when $\eta\gamma = 1$ as (i) all general-equilibrium effects under both price-setting and quantity-setting cancel out and (ii) the partial-equilibrium effects are of the same size under both regimes. Suppose now that η falls so that $\eta\gamma < 1$, quantity-setting firms respond less strongly to productivity shocks while the partial-equilibrium effect (which equals the total effect) under price-setting remains the same. Thus, the economy responds relatively more aggressively to productivity shocks under price-setting. In the $\eta\gamma > 1$ case, the intuition reverses and the economy responds relatively less aggressively to productivity shocks under price-setting.

4.3 Price-Setting and Quantity-Setting Equilibria

So far, we have established the properties of *temporary equilibria* (Definition 1) in which we assume that firms always set prices or quantities. We now study when temporary equilibria are equilibria. In particular, under what conditions do all firms want to set prices (or quantities) when all other firms are doing the same?

Equilibrium Incentives for the "Choice of Choices". To determine the equilibrium incentives for price-setting and quantity-setting, we first combine Lemma 1 with Proposition 1 (along with the fact that consumption is log-linear under both price-setting and quantity-setting from Propositions 2 and 3) to derive that:

$$\Delta_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_\vartheta^2 + \frac{1}{\eta}\left(1-\eta\gamma\right)^2\sigma_{C,t}^2 - \eta\sigma_t^{M^2} + 2(1-\eta\gamma)\sigma_{C,A,t}\right) \tag{49}$$

This reduces the equilibrium determination of Δ_t to a question of how the volatility of consumption and the covariance between consumption and productivity differ across the price-setting and quantity-setting temporary equilibria. We next use our characterizations of temporary equilibrium under price- and quantity-setting (Propositions 2 and 3) to determine the volatility of consumption and its covariance with productivity in each case. This allows us to state the following result which expresses Δ_t as a function of primitives under price-setting and quantity-setting:

Lemma 2 (Prices *vs.* Quantities in Equilibrium)**.** *If all firms set quantities, then the com-*

parative advantage of prices over quantities is given by:

$$\Delta_t^Q = \frac{1}{2}(\eta - 1)\Bigg(\frac{1}{\eta}\sigma_\vartheta^2 - \eta\kappa_t^M\sigma_{M,s}^2 + \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)} + 2\right)(1-\eta\gamma)\frac{\eta\left(\kappa_t^A\right)^2}{1-\kappa_t^A(1-\eta\gamma)}\sigma_{A,s}^2\Bigg) \tag{50}$$

Moreover, all firms can set quantities in equilibrium at time t if and only if $\Delta_t^Q \leq 0$. If all firms set prices, then the comparative advantage of prices over quantities is given by:

$$\Delta_t^P = \frac{1}{2}(\eta - 1)\Bigg(\frac{1}{\eta}\sigma_\vartheta^2 + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2\left(\frac{1-\kappa_t^M}{\gamma}\right)^2\right)\kappa_t^M\sigma_{M,s}^2 + \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_t^A}{\gamma} + 2\right)(1-\eta\gamma)\frac{\left(\kappa_t^A\right)^2}{\gamma}\sigma_{A,s}^2\Bigg) \tag{51}$$

Moreover, all firms can set prices in equilibrium at time t if and only if $\Delta_t^P \geq 0$.

Proof. See Appendix A.5. □

Importantly, since $\Delta_t^Q \neq \Delta_t^P$ in general, others' choice of whether to set prices or quantities affects any given firm's incentives to set prices or quantities. Does the fact that others set prices (quantities) increase or decrease my own desire to set prices (quantities)? Strikingly, we find that these decisions are always *strategic complements*:

Proposition 4 (Complementarity in Choices of Choices)**.** *The decision to set a price or a quantity is one of strategic complements, i.e, $\Delta_t^P \geq \Delta_t^Q$, with strict inequality whenever $\eta\gamma \neq 1$.*

Proof. See Appendix A.6. □

We first consider the case when $\eta\gamma < 1$. In this case, consumption responds more to productivity shocks under price-setting (Corollary 2). Moreover, regardless of the value of $\eta\gamma$, consumption responds more to monetary shocks under price-setting (Corollary 1). Therefore, others being price-setters increases both the variance of consumption and the covariance of consumption with productivity. Both of these forces favor price-setting, as shown in Equation 49. In summary, others setting prices induces aggregate volatility which makes it more attractive for any given firm to also set a price.

In the case of $\eta\gamma \geq 1$, consumption is more responsive to monetary shocks but less responsive to productivity shocks under price-setting versus quantity-setting. In the proof, we show how these effects net out in Equation 49 in the direction of making price-setting more attractive when other firms set prices.

Existence of Price- and Quantity-Setting Equilibria. An immediate corollary of the strategic complementarity uncovered in Proposition 4 is that there always exists at least one "pure" price- or quantity-setting equilibrium:

Corollary 3 (Existence of Pure Equilibria)**.** *There exists an equilibrium.*

This result follows immediately from Proposition 4. In particular, there are two possible cases. First, suppose that firms prefer to set prices if others set quantities, or $\Delta_t^Q \geq 0$. In this case, they even more sharply prefer to set prices if others set prices, or $\Delta_t^P \geq \Delta_t^Q \geq 0$. Therefore, there exists a price-setting equilibrium. Conversely, suppose that firms prefer to quantities when others set quantities, $\Delta_t^Q < 0$. In this case, a quantity-setting equilibrium trivially exists. As these cases are exhaustive, a pure equilibrium exists. We argue that this result is not obvious *ex ante*. In particular, if the decision to set prices was one of substitutes, pure equilibria could fail to exist (of course a mixed equilibrium would always exist).

Characterizing the Presence of Price- and Quantity-Setting Equilibria. The discussion above leaves open the possibility that firms could prefer to set prices when others set prices *and* prefer to set quantities when others set quantities. That is, $\Delta_t^P \geq 0 \geq \Delta_t^Q$. Intuitively, this hinges on the quantitative magnitude of the forces underlying Proposition 4. In the following result, we provide necessary and sufficient conditions in terms of primitives for the coexistence of both equilibria:

Proposition 5 (Characterization of Price- and Quantity-Setting Equilibria)**.** *The following statements are true:*

1. *If $\eta\gamma = 1$, then $\Delta_t^Q = \Delta_t^P = \Delta^Q(0) = \frac{1}{\eta}\sigma_\vartheta^2 - \eta\kappa^M\sigma_{M,s}^2$. Thus, there exists a quantity-setting equilibrium if and only if $\Delta^Q(0) \leq 0$ and there exists a price-setting equilibrium if and only if $\Delta^Q(0) \geq 0$.*
2. *If $\eta\gamma < 1$, then Δ_t^Q and Δ_t^P are strictly increasing in κ_{t-1}^A (decreasing in σ_{t-1}^A). Thus, there exist unique thresholds $\bar{\kappa}_Q^A \in [0,1]$ and $\bar{\kappa}_P^A \in [0,1]$, such that there exists a quantity-setting equilibrium if and only if $\kappa_{t-1}^A \leq \bar{\kappa}_Q^A$ and there exists a price-setting equilibrium if and only if $\kappa_{t-1}^A \geq \bar{\kappa}_P^A$. Moreover, $\bar{\kappa}_Q^A \geq \bar{\kappa}_P^A$.*
3. *If $\eta\gamma > 1$, then Δ_t^Q and Δ_t^P are strictly decreasing in κ_{t-1}^A (increasing in σ_{t-1}^A). Thus, there exist unique thresholds $\bar{\kappa}_Q^A \in [0,1]$ and $\bar{\kappa}_P^A \in [0,1]$, such that there exists a quantity-setting equilibrium if and only if $\kappa_{t-1}^A \geq \bar{\kappa}_Q^A$ and there exists a price-setting equilibrium if and only if $\kappa_{t-1}^A \leq \bar{\kappa}_P^A$. Moreover, $\bar{\kappa}_Q^A \leq \bar{\kappa}_P^A$.*

Proof. See Appendix A.8. □

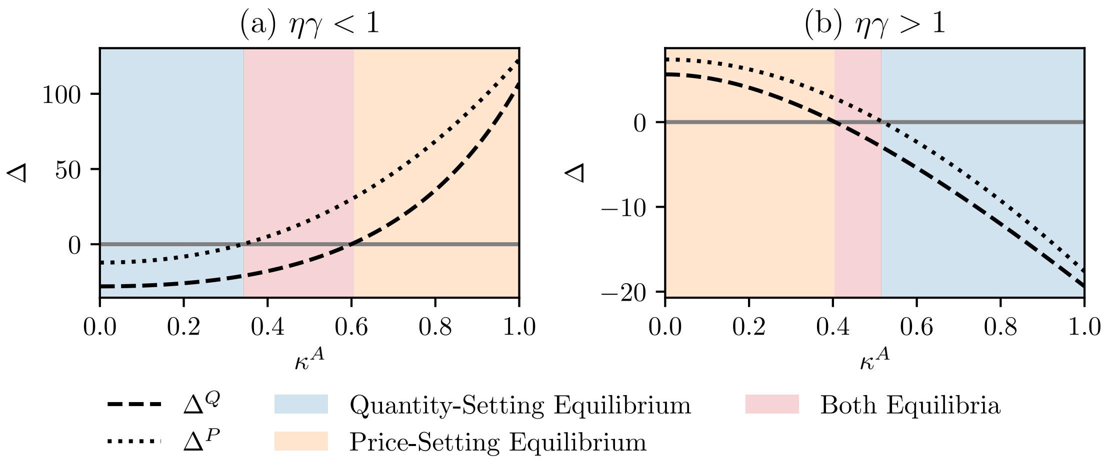
We illustrate cases 2 and 3 graphically in Figure 1. In each case, for example parameter values, we plot Δ^Q and Δ^P as a function of κ^A, holding fixed all other parameters. We identify the points at which each curve cross $\Delta = 0$ and shade the parameter regimes in which there is only a quantity-setting equilibrium, only a price-setting-equilibrium, and both equilibria.

To understand this result, we discuss each case. First, when $\eta\gamma = 1$, there is no strategic complementarity in the choice of what to choose. Thus, Δ is the same under both price-setting and quantity-setting. Moreover, when $\eta\gamma = 1$, the stochastic properties of consumption do not affect Δ. Thus, the existence of a price-setting equilibrium (or a quantity-setting equilibrium) boils down to simply asking if idiosyncratic demand shocks are sufficiently larger than firms' uncertainty about the money supply (and resulting price volatility). When $\eta\gamma < 1$, there is strict strategic complementarity in the choice of what to choose. Moreover, when firms set prices, the economy responds more to productivity shocks under price-setting. Thus, the relative response of the economy to productivity to productivity shocks under price-setting *vs.* quantity-setting is increasing in the precision of agents' knowledge about productivity κ^A_{t-1}. Hence, as κ^A_{t-1} increases, the induced increase in consumption volatility favors price-setting, which increases both Δ^P_t and Δ^Q_t. When $\eta\gamma > 1$, there remains strategic complementarity, but the relative response of the economy to productivity shocks under price-setting *vs.* quantity-setting is *decreasing* in the precision of agents' knowledge about productivity κ^A_{t-1}. Thus, Δ^P_t and Δ^Q_t both decrease in the precision of agents' information about productivity.

This result implies that the economy can dynamically transition between price-setting and quantity-setting regimes for both exogenous and endogenous reasons. First, as aggregate productivity volatility moves exogenously, the economy can transition between times in which uncertainty favors quantity-setting (the blue regions in Figure 1) and times in which uncertainty favors price-setting (the orange regions in Figure 1).

Second, price-setting and quantity-setting can be entirely self-fulfilling in equilibrium (the red regions in Figure 1) and generate endogenous regime shifts. At a basic level, this result stems from the fact that quantity-setting and price-setting can generate macroeconomic volatility that is self-fulfilling. This occurs when the differences in macroeconomic dynamics between the two regimes are sufficiently large to tip incentives in favor of one or the other. A particularly extreme manifestation of this phenomenon is that if the economy lies in the multiplicity region, then we can construct equilibria that switch *arbitrarily* between price- and quantity-setting from period to period. This is formalized below:

Figure 1: Example Characterization of Price- and Quantity-Setting Equilibria



Note: This figure illustrates Proposition 5 under example parameter values. In each panel, we plot Δ^Q (dashed line) and Δ^P (dotted line) as a function of κ^A, fixing all other parameter values. In Example A, we use parameters such that $\eta\gamma < 1$ (case 2 of Proposition 5). In Example B, we use parameters such that $\eta\gamma > 1$ (case 2 of Proposition 5). We shade the region with only a quantity-setting equilibrium blue, the region with only a price-setting equilibrium orange, and the region with both equilibria red.

Condition 1 (Regime-Switching)**.** *Suppose one of the following three statements is true:*

1. $\eta\gamma = 1$ *and* $\frac{1}{\eta}\sigma_\vartheta^2 = \eta\kappa^M\sigma_{M,s}^2$
2. $\eta\gamma < 1$ *and* $\kappa_{t-1}^A \in [\bar{\kappa}_P^A, \bar{\kappa}_Q^A]$ *for all* $t \in \mathbb{N}$
3. $\eta\gamma > 1$ *and* $\kappa_{t-1}^A \in [\bar{\kappa}_Q^A, \bar{\kappa}_P^A,]$ *for all* $t \in \mathbb{N}$

Corollary 4 (Regime-Switching)**.** *Consider any partition of* $\mathbb{N}$ *into two sets* $\mathcal{T}^P$ *and* $\mathcal{T}^Q$*. Under (and only under) Condition 1, there exists an equilibrium such that firms set quantities in all periods* $t \in \mathcal{T}^Q$ *and firms set prices in all period* $t \in \mathcal{T}^P$*.*

Thus, our analysis offers a fully endogenous explanation for the stochastic volatility patterns observed in macroeconomic data and, by extension, in our calculation for the relative incentives for price-setting and quantity-setting in Section 6. Moreover, in contrast to the standard macroeconomics literature (see *e.g.,* Berger and Vavra, 2019), there is no dichotomy between shocks and responsiveness in our setting: the volatility of shocks determines (and is determined by) the macroeconomic regime. Hence, there is a two-way relationship between the volatility of shocks and the responsiveness of the economy to shocks.

5 Monetary Policy Transmission

In our equilibrium analysis, we highlighted how the transmission of money supply shocks to aggregate quantities and prices was shaped by firms' price *vs.* quantity choice. But our model allowed no role for systematic monetary *policy*, or manipulation of the money supply in response to economic conditions. We now investigate how monetary stabilization policy is affected by firms' choices of what to choose. We derive how the identity of the decision regime (price-setting or quantity-setting) affects the transmission of policy and how policy affects firms' choice of decision variable. Taken together, these results suggest novel trade-offs for policymakers who wish to both manage output and price variation *within* a regime and, potentially, induce the economy to transition to a more advantageous regime.

5.1 Set-Up: The Model with a Monetary Rule

To study monetary policy in our model, we allow the money supply to have a drift that depends linearly on aggregate productivity:

$$\log M_t = \log M_{t-1} + \mu_M + \alpha_A \log A_t + \sigma_m \log m_t \tag{52}$$

The "policy instruments" are μ_M and α_A. The former controls average money growth and the latter controls responses to aggregate conditions. Intuitively, $\alpha_A > 0$ corresponds to "leaning with" shocks and $\alpha_A < 0$ corresponds to "leaning against" shocks. The term $\log m_t \sim N(0, 1)$ is an (uncontrolled) monetary shock and is IID across time. Finally, for this analysis, we assume that $\log A_t \sim N(\mu_A, \sigma_A^2)$ and is IID. The definitions of temporary equilibrium and equilibrium are analogous to those given in Section 3, with Equation 52 replacing the original money-supply evolution in Equation 21. The original model, under the restriction to IID productivity, is nested when $\alpha_A = 0$ and $\log m_t = \epsilon_t^M$.

5.2 Monetary Rules and Aggregate Outcomes

We begin by characterizing equilibrium dynamics under price- and quantity-setting temporary equilibria under the policy rule:

Proposition 6 (Outcomes under the Monetary Rule)**.** *If all firms set prices, output in the unique log-linear equilibrium follows:*

$$\log C_t = \chi_{0,t-1}^{T,1} + \frac{1}{\gamma}\left(\alpha_A(1-\kappa^A) + \kappa_A\right)\log A_t + \frac{1}{\gamma}\sigma_m(1-\kappa^m)\log m_t \tag{53}$$

and the aggregate price in the unique log-linear equilibrium follows:

$$\log P_t = \tilde{\chi}^{T,1}_{0,t-1} + (\alpha_A - 1)\kappa^A \log A_t + \sigma_m \kappa^m \log m_t \tag{54}$$

If all firms set quantities, output in the unique log-linear equilibrium follows:

$$\log C_t = \chi^{T,2}_{0,t-1} + \frac{\eta\kappa^A}{1-\kappa^A(1-\eta\gamma)} \log A_t \tag{55}$$

and the aggregate price in the unique log-linear equilibrium follows:

$$\log P_t = \tilde{\chi}^{T,2}_{0,t-1} + \left(\alpha_A - \frac{\eta\gamma\kappa^A}{1-\kappa^A(1-\eta\gamma)}\right) \log A_t + \sigma_m \log m_t \tag{56}$$

where $\chi^T_{0,t-1}$ and $\tilde{\chi}^T_{0,t-1}$ are constants that depend only on parameters and past shocks to the economy.

Proof. See Appendix A.9 □

Under price-setting, the response of both consumption and prices to productivity shocks increases in α_A. This is natural since a policymaker setting $\alpha_A > 0$ induces demand when productivity is high, and a policymaker setting $\alpha_A < 0$ cools off demand when productivity is high. When firms set quantities, equilibrium consumption is invariant to α_A and equilibrium prices are proportional to the money supply expansion $\alpha_A \log A_t$. This follows from the neutrality of money under quantity-setting (Proposition 3). Finally, manipulating the drift of the money supply affects the aggregate price level, but neither the level of consumption nor the responsiveness of consumption and prices to shocks.

5.3 How Monetary Rules Affect Choices of Choices

The previous result established how monetary policy affected the economy in a fixed regime, price-setting or quantity-setting. We now study how the possibility of each regime is itself shaped by policy.

Monetary Policy and Quantity-Setting. First, even though monetary policy does not affect real aggregate outcomes under quantity-setting, by affecting aggregate price-volatility it still affects the possibility that the economy is in a quantity-setting regime. To study these effects, we say that *quantity-setting is more possible* if Δ^Q, the relative preference for price-setting conditional on all other firms setting quantities, decreases. The following result characterizes how the nature of monetary policy affects the possibility of quantity-setting:

Proposition 7 (Monetary Policy and the Possibility of Quantity-Setting)**.** *Increasing α_A makes quantity-setting less possible if $\alpha_A < 1$ and more possible if $\alpha_A > 1$. Moreover, relative to passive monetary policy ($\alpha_A = 0$), active monetary policy makes quantity-setting less possible if and only if $\alpha_A \in (0, 2)$.*

Proof. See Appendix A.10. $\square$

Under a quantity-setting regime, monetary policy affects only aggregate prices and not aggregate quantities. When policy is neutral, or $\alpha_A = 0$, prices move opposite to productivity. Increasing α_A away from 0 has two effects. First, it increases the volatility of the money supply, which increases the volatility of prices, favoring quantity-setting. This effect is second-order in α_A. Second, it mechanically increases the covariance between productivity shocks and the money supply, which reduces the covariance between prices and real marginal costs, favoring price-setting. This effect is first-order in α_A. Further, the second effect dominates the first effect until α_A reaches one, at which point the quadratic nature of the first effect dominates. Thus, monetary policy that leans against the wind ($\alpha_A < 0$) makes quantity-setting more likely, monetary policy that moderately leans into the wind ($\alpha_A \in (0, 2)$) makes quantity-setting less likely, and monetary policy that aggressively leans into the wind ($\alpha_A > 2$) makes quantity-setting more likely.

Monetary Policy and Price-Setting. We now study how monetary policy affects the possibility of price-setting equilibria. We say that price-setting is more possible if Δ^P, the relative preference for price-setting conditional on all other firms setting prices, increases. In this case, monetary policy has the same direct effects on the relative preference for prices or quantities through the volatility of money and the covariance between money and productivity as under quantity-setting. However, monetary policy now also has indirect, equilibrium effects by shaping how productivity shocks transmit into real output.

Proposition 8 (Monetary Policy and the Possibility of Price-Setting)**.** *Starting from passive monetary policy, increasing α_A makes price-setting more possible, i.e.,*

$$\frac{\partial}{\partial \alpha_A} \Delta^P |_{\alpha_A = 0} > 0 \tag{57}$$

Moreover, Δ^P is a strictly concave function of α_A if and only if $\eta\gamma > \frac{1-\kappa^A}{2-\kappa^A} \in (0, \frac{1}{2})$. When Δ^P is concave, increasing α_A makes price-setting more possible if $\alpha_A < \alpha_A^*$ and less possible *if* $\alpha_A > \alpha_A^*$*, where:*

$$\alpha_A^* = \frac{1 + \left(\frac{1-\eta\gamma}{\eta\gamma}\right)^2 (1-\kappa^A)\kappa^A + \frac{1-\eta\gamma}{\eta\gamma}(1-\kappa^A)}{1 - \left(\frac{1-\eta\gamma}{\eta\gamma}\right)^2 (1-\kappa^A)^2} > 0 \tag{58}$$

When Δ^P *is strictly convex, increasing* α_A *makes price-setting more possible if* $\alpha_A > \alpha_A^*$ *(where* $\alpha_A^* < 0$*) and less possible if* $\alpha_A < \alpha_A^*$*.*

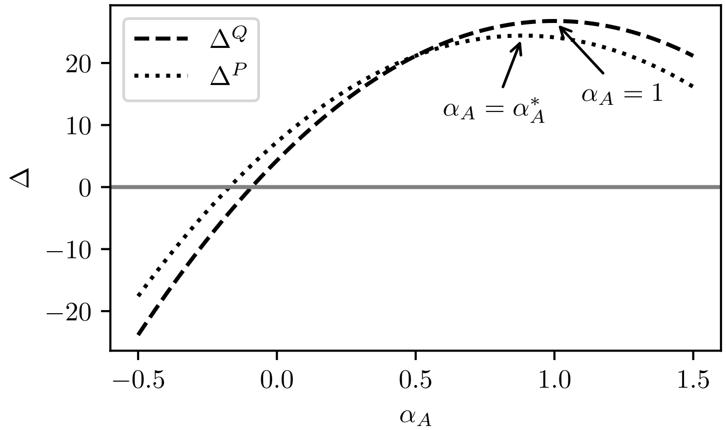
Proof. See Appendix A.11 □

Under a price-setting regime, increasing α_A has the same direct effects on the volatility of the money supply (that favor quantity-setting to second-order) and the covariance between money and productivity (that favor price-setting to first-order). In addition, however, there are now indirect effects as equilibrium consumption in a price-setting regime becomes more volatile when monetary policy leans into productivity (a second-order effect that favors price-setting) and has a higher covariance with productivity (a first-order effect that favors price-setting when $\eta\gamma < 1$ and favors quantity-setting when $\eta\gamma > 1$). The first-order effects always net in the direction of favoring price-setting and so policy that leans into productivity shocks makes price-setting more likely (see Figure 2). The relative magnitude of the second-order effects is ambiguous and the direct effects dominate the indirect effects if and only if substitutability $\eta\gamma$ is sufficiently high (a sufficient condition is that $\eta\gamma > \frac{1}{2}$). In this case, price-setting is made most likely by a policy that leans into productivity shocks at rate α_A^*. In the convex case, the policymaker can always make Δ^P as large as they like by setting α_A large enough in absolute value.

A New Trade-Off for Output Stabilization Policy. If a policymaker wishes to stabilize output, they can do so by setting $\alpha_A < 0$, thereby leaning against productivity shocks. However, this will only succeed if the economy remains in a price-setting regime. Propositions 7 and 8 show that setting $\alpha_A < 0$ always reduces Δ^Q and always locally reduces Δ^P, as in Figure 2 (it also does so globally whenever Δ^P is concave or whenever $\alpha_A > \alpha_A^*$ when Δ^P is convex). Thus, by stabilizing output, they both make price-setting harder to sustain and quantity-setting easier to sustain. In this sense, attempts to stabilize output have the potential to switch the economy into a quantity-setting regime.

When the economy is less responsive to productivity under quantity-setting ($\eta\gamma < 1$), this switch is desirable as output is less volatile under quantity-setting. Thus, the policymaker has a free lunch: output stabilization policies stabilize output conditional on remaining in a price-setting regime and may switch the economy into a less volatile quantity-setting regime. However, when the economy is more responsive to productivity shocks under price-setting

Figure 2: The Effect of Policy on the Comparative Advantage of Prices



Note: This figure illustrates Propositions 7 and 8 under example parameter values. It plots Δ^Q (dotted line) and Δ^P (dashed line) as a function of the policy parameter α_A, fixing all other parameters. We assume $\eta\gamma > \frac{1}{2}$, so that Δ^P is concave. Both Δ^Q and Δ^P are locally increasing at $\alpha_A = 0$, and are maximized at $\alpha_A = 1$ and $\alpha_A = \alpha_A^*$, respectively.

($\eta\gamma > 1$), this switch is undesirable, as output is more volatile under quantity-setting. Hence, attempts to stabilize output can backfire by inducing an adverse regime shift.

A New Trade-Off for Price Stabilization Policy. If a policymaker wishes to stabilize the price level, they can do so by setting $\alpha_A > 0$ and leaning into productivity shocks. This is true in both price-setting and quantity-setting regimes. In a price-setting regime, this has the familiar cost of increasing output volatility and induces a trade-off between lowering the price level and keeping real output high. However, in a quantity-setting regime, the policymaker has a free lunch: they can control the price-level with monetary policy while the real economy remains unaffected. In other words, our model generates a state-dependent "Phillips curve," which is shaped by the nature of policy as well as microeconomic and macroeconomic uncertainty. In particular, policymakers face a "Phillips curve" if and only if the economy lies in a price-setting regime.

Regime Switching and Monetary Policy. The preceding discussion suggests that monetary policy rule can induce regime-switches in the economy, by reducing Δ^P or increasing Δ^Q. With active monetary policy, however, it is possible for firm decisions to set price or quantities can be *strategic substitutes*. Therefore, a monetary policy rule that makes Δ^P negative may not induce a "pure" quantity setting regime if $\Delta^Q \geq 0 \geq \Delta^P$. The next proposition gives sufficient conditions for planning choices to be strategic complements, thereby ensuring the existence of at least one "pure" quantity or price setting equilibrium.

Proposition 9 (Monetary Policy and Regime Switching)**.** *The decision to set a price or a quantity is one of strategic complements, i.e.* $\Delta^P \geq \Delta^Q$*, if one of the following conditions are satisfied:*

1. $\eta\gamma = 1$
2. $\eta\gamma < 1$ *and* $\alpha_A \geq \tilde{\alpha}_A$*, where*

$$\tilde{\alpha}_A \equiv \frac{-\kappa^A(1-\eta\gamma)}{1-\kappa^A(1-\eta\gamma)} \in (-\infty, 0) \tag{59}$$

3. $\eta\gamma > 1$ *and* $\alpha_A \leq \tilde{\alpha}_A$*, where*

$$\tilde{\alpha}_A \equiv \frac{-\kappa^A(1-\eta\gamma)}{1-\kappa^A(1-\eta\gamma)} \in (0, 1) \tag{60}$$

Moreover,

$$\lim_{\alpha_A \to \pm\infty} \left(\Delta^P - \Delta^Q\right) \geq 0 \tag{61}$$

Proof. See Appendix A.12. □

In particular, planning choices are always strategic complements if $\eta\gamma < 1$ and α_A is not too negative, or $\eta\gamma > 1$ and α_A is not too positive. The intuition for this result is as follows. If $\eta\gamma < 1$, a larger covariance between consumption and productivity, $\sigma_{C,A}$, enters positively into (16). This is because increasing $\sigma_{C,A}$ reduces $\sigma_{\Psi,\mathcal{M}}$ more than it raises $\eta \times \sigma_{P,\mathcal{M}}$ whenever $\eta\gamma < 1$. However, $\sigma_{C,A}$ is larger under price setting than quantity setting if and only if $\alpha_A \geq \tilde{\alpha}_A$. If $\eta\gamma > 1$, $\sigma_{C,A}$ enters negatively into (16), which requires that $\sigma_{C,A}$ be lower under price setting than quantity setting to ensure the existence of complementarities. This is occurs if and only if $\sigma_{C,A} \leq \tilde{\alpha}_A$, as per the proposition.

Although the presence of strategic substitutability implies that a "pure" equilibrium may not exist for some parameter values, a mixed equilibrium clearly exists. Appendix C.3 characterizes – to first-order – the dynamics of the price level and consumption in the presence of mixing, in which $\lambda_t \in (0, 1)$ fraction of firms set prices, and a fraction $1 - \lambda_t$ of firms set quantities. In the case with mixing, the dynamics of the economy become a convex combination of the "pure" dynamics under price setting and quantity setting.

6 Price and Quantity Regimes in US Data

Having described the theoretical model and its equilibrium implications for macroeconomic dynamics and policy, we now turn to the data. We first ask: does the data, when viewed through the lens of our model, suggest that firms would prefer to set prices or quantities in different realistic circumstances? Or does the price-*vs.*-quantities choice decidedly favor one over the other all the time?

In this section, we calculate the relative advantage of price-setting from Proposition 1 in US data. Our approach is to combine time-varying statistical estimates of each volatility term in Equation 16 with an external calibration for the demand elasticity. We find that price-setting is optimal in times of tame inflation (the Great Moderation) and high demand uncertainty (the Great Recession or first quarter of Covid-19 lockdown), while quantity-setting is optimal in times of volatile inflation (the 1970s and post-Covid inflation). Thus, the economic considerations in Proposition 1 deliver a close "horse race," with different winners in different periods of history.

6.1 Data and Methods

For our main calculation, we use quarterly-frequency US data on real GDP, GDP deflator, and capacity-utilization adjusted total factor productivity (TFP) (Basu et al., 2006; Fernald, 2014) from 1960Q1 to 2022 Q4. Thus, our mapping from model to data considers quarterly-frequency decisions.

We map these variables to model quantities as follows. First, consistent with our equilibrium model, we model the demand shock as $\Psi = Y\vartheta$, where Y is aggregate real GDP (i.e., "aggregate demand") and ϑ is a firm-specific demand shock that is, by construction, orthogonal to aggregate conditions. Thus, we can decompose $\sigma_\Psi^2 = \sigma_Y^2 + \sigma_\vartheta^2$, where the latter two terms are respectively the variances of $\log Y$ and $\log \vartheta$.

Second, we assume as in the equilibrium model that real marginal costs are determined at the aggregate level as $\mathcal{M} = Y^\gamma / A$, where $\gamma > 0$ measures wealth effects in labor supply and controls the cyclicality of real wages and A is an aggregate shock to productivity. We set $\gamma = 0.095$ based on the calibration in Flynn and Sastry (2022a) for the rigidity of US real wages. We measure A via the aforementioned data on the utilization-adjusted aggregate Solow residual. The assumption that physical productivity is identical across firms, while demand varies, is consistent with the findings of Foster et al. (2008) that cross-firm variation in revenue total factor productivity (TFPR) derives almost exclusively from demand differences rather than marginal cost differences within specific industries. Assuming that all cross-firm variation derives from demand shocks biases our calculation toward price-setting, in light of our findings in Proposition 1.

Finally, we assume that uncertainty about idiosyncratic demand is directly proportional to uncertainty about aggregate marginal costs, or $\sigma_\vartheta^2 = R^2 \sigma_{\mathcal{M}}^2$. We justify this based on the finding of Bloom et al. (2018) that the stochastic volatility of TFPR among manufacturing firms ("micro volatility") is well modeled as directly proportional to stochastic volatility in aggregate conditions ("macro volatility"). Based on these authors' quantitative findings, we take $R = 6.5$ as a baseline. In an extension, we directly use (annual) data on TFPR dispersion from Bloom et al. (2018) to inform our calculation.

The assumptions described above make all variance terms in Proposition 1 functions of the time-varying uncertainties about aggregate real GDP, inflation, and real marginal costs. We estimate these time-varying uncertainties using a multivariate GARCH model. In particular, letting Z_t denote the vector of these three variables, we model

$$Z_t = AZ_{t-1} + \varepsilon_t, \qquad \varepsilon \sim N(0, \Sigma_t), \qquad \Sigma_t = D_t^{\frac{1}{2}} R D_t^{\frac{1}{2}} \tag{62}$$

where A is a matrix of AR(1) coefficients, D_t is a diagonal matrix of time-varying variances (and $D_t^{\frac{1}{2}}$ is a diagonal matrix of standard deviations) and R is a static matrix of correlations. We assume that each diagonal element of D_t, denoted as $\sigma_{i,t}^2$, evolves as $\sigma_{i,t}^2 = s_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$, with unknown constant s_i and coefficients (α_i, β_i). Formally, this is a GARCH(1,1) model with constant conditional correlations (Bollerslev, 1990). In our context, the restriction to constant correlations restricts the covariances in Equation 16 to move in proportion to the variances and thus rules out the possibility that the correlation structure among output, prices, and marginal costs varies over time. We estimate all of the parameters via joint maximum likelihood.

From this, we derive maximum-likelihood point estimates of every element of Σ_t, which correspond to the variances in the (Gaussian) conditional forecast of Z_t. Letting $\hat{\sigma}_t$ denote the point estimates of specific elements of that matrix, we compute:

$$\hat{\sigma}_{\Psi,t}^2 = \hat{\sigma}_{Y,t}^2 + R^2 \hat{\sigma}_{\mathcal{M},t}^2 , \quad \hat{\sigma}_{\Psi,\mathcal{M},t} = \hat{\sigma}_{Y,\mathcal{M},t} \tag{63}$$

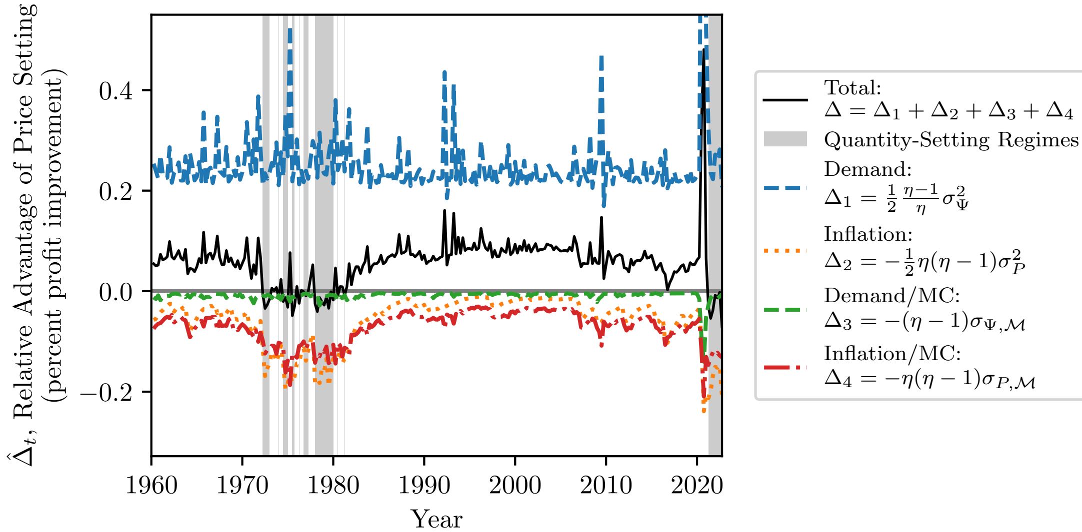
Finally, we take a central estimate of $\eta = 9$ from the study of Broda and Weinstein (2006). These authors use comprehensive panel data on US imports to estimate demand curves at the level of disaggregated products. This is, usefully for our purposes, *direct* evidence for the slope of demand curves, as opposed to indirect evidence from matching average product markups under the assumption that firms are full-information price setters.

We now calculate our empirical proxy for the benefit of price-setting,

$$\hat{\Delta}_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\hat{\sigma}_{\Psi,t}^2 - \eta \hat{\sigma}_{P,t}^2 - 2\hat{\sigma}_{\Psi,\mathcal{M},t} - 2\eta \hat{\sigma}_{P,\mathcal{M},t}\right) \tag{64}$$

Our calculation captures uncertainty about outcomes realized in quarter t, and is measurable

Figure 3: The Relative Benefit of Price-Setting in US Data



Note: This figure plots our empirical estimate of $\hat{\Delta}_t$ (the comparative advantage of price-setting relative to quantity-setting) and its components, as defined in Proposition 1 (Equation 16). The black line plots $\hat{\Delta}_t$, in units of expected percent profit improvement (100 times log points). The blue (dashed), orange (dotted), green (dashed), and red (dash-dotted) lines plot each of the four components of $\hat{\Delta}_t$, corresponding to uncertainty about different variables. The grey shading denotes periods in which $\hat{\Delta}_t < 0$ and thus, according to Proposition 1, quantity-setting is optimal for firms. As described in Section 6.1, the calculation uses estimates of time-varying volatilities from a CCC GARCH(1,1) model and a calibrated demand elasticity of $\eta = 9$. The demand component exceeds the scale of the figure in Q2 and Q3 of 2020.

in data from quarter $t-1$ and prior. It therefore describes incentives of a decisionmaker fixing a choice for quarter t based on their uncertainty at the end of quarter $t-1$.

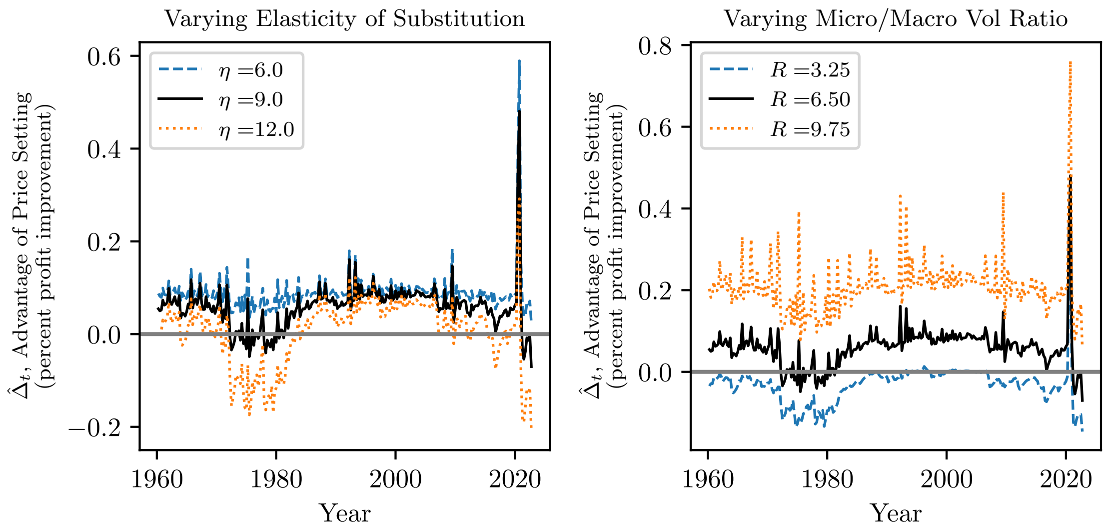
6.2 Results: Quantity-Setting Regimes Emerge When Inflation is Volatile

We plot our calculation of $\hat{\Delta}_t$ in Figure 3. We show our overall calculation in black and each component in colors. We shade periods which favor quantity-setting, or for which $\hat{\Delta}_t < 0$.

Strikingly, both quantity- and price-setting are optimal at different points in the sample. Thus, viewed through the lens of our model and its mapping to the data, firms may be either price- or quantity-setters depending on the macroeconomic context. Moreover, through the same lens, this evidence rules out the conventional assumption that firms always choose prices or always choose quantities.

Price-setting is optimal in most of the sample, or 219 of 251 quarters. This notably

Figure 4: The Relative Benefit of Price-Setting Under Alternative Parameters



Note: Both panels plot our empirical estimate of $\hat{\Delta}_t$ defined in Proposition 1 (Equation 16) under alternative assumptions for the elasticity of substitution η (left) and the micro-to-macro volatility ratio R (right). In both plots, our baseline estimate corresponds to the solid black line.

comprises the 1960s and the Great Moderation, in which both inflation and demand variance were relatively tame, and the Great Recession and onset of the Covid-19 Lockdown Recession (Q2 2020), when demand variance abruptly spiked.

Quantity-setting is optimal intermittently between 1972 Q2 and 1981 Q2, for a total of 25 of the possible 37 quarters in this period, and continuously between 2021 Q2 and the end of the sample. These all correspond to periods of particularly high contributions of the terms corresponding to inflation variance and inflation-marginal-cost covariance. Through the lens of the model, firms would prefer to set quantities in these periods to hedge against the increase in uncertainty about joint movements in inflation and marginal costs. Our calculation weighs this consideration against demand risk, which favors price-setting and may also be elevated in times of (real) crisis. For example, in 1975 Q2 and 2021 Q1, demand uncertainty is sufficiently high to outweigh elevated inflation and inflation-marginal-cost uncertainty, and our calculation favors price-setting on net ($\hat{\Delta}_t > 0$).

We finally note that the relative advantage of one method over another is always relatively small in payoff terms. In our sample, this advantage peaks at 0.48% (0.0048 log points) in Q3 of 2020. In all periods excluding Q2 and Q3 of 2020, the difference peaks at 0.16%. This is a striking juxtaposition with the model prediction that a change in firm behavior between price- and quantity-setting can have large effects on equilibrium outcomes.

Robustness to Parameter Values and Measurement Strategies. Two parameters that were central to our calculation, but difficult to pin down in the data, were the price elasticity of demand ($\eta = 9$) and ratio of micro to macro volatility ($R = 6.5$). In Figure 4, we plot the implied time series for Δ under specific alternative assumptions for each parameter. In Appendix Figure 6, we vary both parameters continuously over a larger grid and plot "heat maps" for the average value of $\hat{\Delta}_t$ and the percentage of the sample with $\hat{\Delta}_t > 0$.

Decreasing the elasticity of demand favors price-setting, while increasing the elasticity of demand favors quantity-setting (left panel). The primary reason, quantitatively, is that highly inelastic demand curves amplify the effects of demand shocks on prices for fixed quantities, and hence increase potential losses from quantity-setting. In the data, this further pushes toward price-setting, especially in time periods with especially high demand volatility.

Increasing demand risk favors price-setting by construction (right panel). In particular, increasing the extent of microeconomic volatility by 50% favors price-setting in all periods (orange dotted line), while decreasing this parameter by 50% implies quantity-setting in a majority of periods (blue dashed line). As noted by Bloom et al. (2018), calibrating this parameter on the basis of observed variances in *measured* firm-level fundamentals requires modeling choices. In particular, one must take a stand on what fraction of measured volatility corresponds to measurement error and what fraction of volatility from an econometrician's perspective is unknown to firm managers, who likely have superior information.

As an alternative strategy to measure the contribution of idiosyncratic volatility, we can use the direct measurements of Bloom et al. (2018) based on annual data from manufacturing establishments from 1972 to 2010, along with assumptions about measurement error and observability of shocks. To accommodate this variant calculation, we re-estimate the VAR(1) CCC GARCH(1,1) model on annual data for the same macro time series. We then use the Bloom et al. (2018) estimates of the cross-sectional standard deviation of manufacturing TFPR along with those authors' quantitative assumption that 45.4% of this measured volatility (standard deviation) corresponds to measurement error. We make the intentionally extreme assumption that all of this remaining variance is unforecastable by firms.[5] Appendix Figure 7 shows our results. This calculation echoes the conclusion that the 1970s were favorable to quantity-setting due to the relatively high inflation volatility and relatively low demand volatility.

Comparison to External Evidence. An alternative way to gauge the plausibility of firms' entertaining both price- and quantity-setting plans is via direct survey evidence. As observed by Reis (2006), Aiginger (1999) collected data on this topic. In a survey of managers of Austrian manufacturing firms, he asked: "What is your main strategic variable: do you

[5] This procedure implies a median R_t of 24.2 over our sample.

decide to produce a specific quantity, thereafter permitting demand to decide upon price conditions, or do you set the price, with competitors and the market determining the quantity sold?" Among managers, 32% said that they use the quantity plan and 68% said that they use price the price plan. We interpret this as additional evidence that neither price nor quantity plans is obviously favored in practice.

7 Testing the Model: Asymmetric Effects of Monetary Policy in Price and Quantity Regimes

Our model predicts that expansionary monetary shocks have muted effects on real output and exaggerated effects on prices in a quantity-setting regime compared to a price-setting regime (Corollary 1). The model also predicts that incentives for price-setting are shaped by the volatility of macroeconomic and microeconomic aggregates in a specific way (Proposition 1 and Lemma 2). Crucially, both predictions rely purely on the premise of "choice of choices" and not on specific parameter restrictions.[6] Thus, we can use them to derive an empirical test of our model's new substantive assumption.

In this final section, we provide suggestive evidence consistent with these predictions. In particular, using local projection regressions, we find that output responds more negatively and price respond less negatively to contractionary Romer and Romer (2004) monetary policy shocks in price-setting regimes relative to quantity-setting regimes, measured using the method of Section 6. Through the lens of our analysis, these results validate that regime shifts between price and quantity-setting shape the transmission of economic shocks.

7.1 Methods

We measure monetary policy shocks using the methodology of Romer and Romer (2004). These authors residualize changes in the Federal Funds Rate on the Federal Reserve's macroeconomic projections reported in the Greenbook. Specifically, we use the updated series reported in Ramey (2016) which spans March 1969 to December 2007. We aggregate these shocks to a quarterly-frequency variable, MonShock_t by summing. The key quantity-setting regimes that overlap with the studied sample of Romer and Romer (2004) shocks are primarily in the 1970s. We translate $\hat{\Delta}_t$ into a binary variable which proxies for whether the economy lies in a price-setting regime, $\text{PriceSet}_t = 1_{\Delta_{t+1}>0}$. In the model, this object determines whether decisionmakers who observe data before and during time t would set prices

[6]By contrast, the sign of the differential response to productivity shocks in each regime (Corollary 2) does depend on parameters.

as their decision variable for period $t+1$. This timing convention is appropriate since we will focus on how macroeconomic aggregates at time $t+1$ and onward respond to shocks at time t. Formally, Corollary 1 implies that the following relationships hold:

$$\log Y_t = \chi_M^P \mathbb{I}[\Delta_t > 0] \log M_t + \varepsilon_t^Y \tag{65}$$

$$\log P_t = \log M_t - \tilde{\chi}_M^P \mathbb{I}[\Delta_t > 0] \log M_t + \varepsilon_t^P \tag{66}$$

where $\chi_M^P > 0$, $\tilde{\chi}_M^P \in (0,1)$, and $\Delta_t, \log M_t, \varepsilon_t^Y$ and ε_t^P are independent random variables.

To estimate an empirical analog of these equations, we proxy for real output with real GDP, the price level with the GDP deflator, and the money supply with the MonShock_t. We estimate the state-dependent response of outcomes $Z_t \in \{\text{RealGDP}_t, \text{GDPDeflator}_t\}$ to the variable MonShock_t by running the following local projection regressions for each horizon $h \in \{1, \dots, 12\}$:

$$Z_{t+h} = \beta_h \cdot \text{MonShock}_t + \gamma_h \cdot \text{PriceSet}_t + \phi_h \cdot (\text{MonShock}_t \times \text{PriceSet}_t) + \tau' X_t + \varepsilon_{t,h} \tag{67}$$

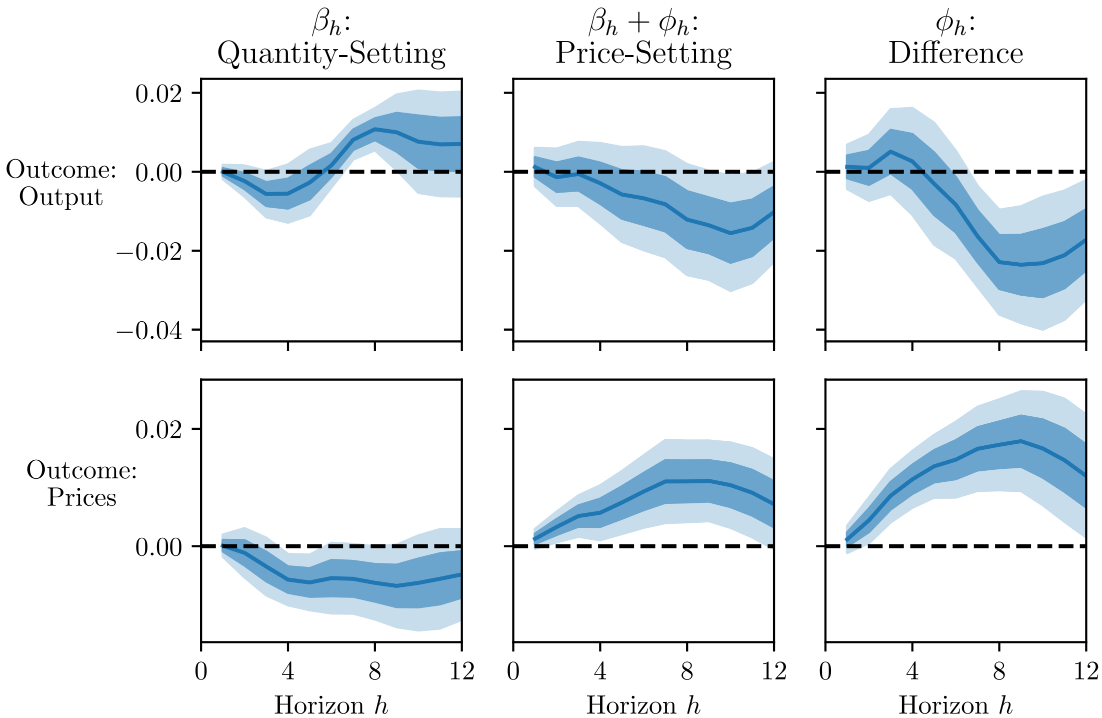
As control variables, we include the contemporaneous and lagged values of real GDP, GDP deflator, and utilization-adjusted TFP, and interactions of all of these variables with PriceSet_t.[7] Including the interaction variables is consistent with our model's implications that the joint dynamics of macroeconomic variables may change between the two regimes.[8] In all reported results, we report frequentist confidence intervals based on Newey and West (1987) standard errors with a six-quarter bandwidth.

The coefficients $\{\beta_h\}_{h=1}^H$ measure the response of output (or prices) to monetary shocks in the quantity regime. We predict that $\beta_h < 0$ for both outcomes. The coefficients $\{\phi_h\}_{h=1}^H$ measure the differential response of output (or prices) to monetary shocks in the price regime, compared to the quantity regime. We predict that $\phi_h < 0$ when real GDP is the outcome and $\phi_h > 0$ when the GDP deflator is the outcome.

[7] As observed by Ramey (2016), including contemporaneous values amounts to assuming a zero contemporaneous response of macroeconomic quantities to shocks on impact, as is typical in the structural VAR literature (e.g., Christiano et al., 2005). This is also consistent with our conditioning on Δ_t, which is measurable in time-t macroeconomic aggregates. Results are very similar when we do not control for contemporaneous values, suggesting that this timing assumption is close to correct in the data.

[8] Tenreyro and Thwaites (2016) make a similar observation about the necessity of these controls in a local-projections estimation of whether monetary policy shocks, also measured as in Romer and Romer (2004), have different effects in recessions.

Figure 5: Choice of Choices Shapes Monetary Policy Transmission



Note: These plots display our estimates of the state-dependent response to monetary policy shocks from Equation 67. The outcome variable is real GDP in the top row and GDP deflator in the bottom row. The columns respectively show our estimates of β_h, the response under quantity-setting; $\beta_h + \phi_h$, the response under price-setting; and ϕ_h, the difference between the price-setting and quantity-setting responses. In each plot, the solid line gives the point estimates, the dark shaded region gives 68% confidence intervals, and the light shaded region gives 95% confidence intervals, where the latter two are based on Newey and West (1987) standard errors with a six-quarter bandwidth.

7.2 Main Results: State-Dependent Effects on Output and Inflation Line Up With the Theory

We show our results graphically in Figure 5. We first consider our results for output (top row). We find on average a zero response to monetary shocks in quantity-setting regimes ($\beta_h = 0$; first column). This average zero response belies weak evidence of a negative response at shorter horizons ($h < 6$) and a positive response at longer horizons ($h \in \{7, 8\}$). By contrast, we find a consistently negative response under price-setting for all horizons $h > 4$. This is statistically significant at the 68% level for $h \geq 6$ and at the 95% level for $h \in \{10, 11\}$. The difference between these responses is also negative ($\phi < 0$) at these longer horizons, and statistically significant at the 95% level for $h \geq 7$. These results, taken together, are consistent with our theory: in price-setting regimes, contractionary monetary policy has considerably more power to shape real outcomes.

We next consider our results for prices (bottom row). We find a small negative response in quantity-setting regimes (column 1) and a significant positive response in price-setting regimes (column 2). The second prediction violates the theory in the direction of the familiar "price puzzle" (see, e.g., Ramey, 2016). But our prediction for the difference of coefficients is consistent with the theory ($\phi_h > 0$, column 3): under quantity-setting regimes, contractionary policy is more able to control the price level.

7.3 Interpretation and Discussion

Comparison to the Literature. An empirical literature draws a mixed conclusion on whether monetary policy is more or less powerful in "downturns," broadly defined. Weise (1999) finds weaker price effects and stronger output effects when output is initially low; Garcia and Schaller (2002) and Lo and Piger (2005) find stronger responses of output in recessions; and Tenreyro and Thwaites (2016) find *weaker* responses of output and prices in recessions. Our analysis differs both because (i) it conditions on a different variable, the model's prediction for whether firms set prices or quantities, which itself depends on uncertainties rather than means; and (ii) it tests for differences in both the response of output to monetary shocks and the response of prices to monetary shocks, as predicted by the theory.

Lessons from History, and for the Present. Interpreting the historical data through the lens of the model, these results suggest that monetary policy actions *of a fixed size* may have had greater effects on inflation in the 1970s and early 1980s, a quantity-setting regime, than in other periods. This notably includes the first contractionary "Volcker shock" in 1979 Q3 as well as the *expansionary* shock in 1980 Q2, when rates surprisingly plummeted.

Although outside the scope of our empirical analysis, the fact that the Volcker Fed's *conquering* of US inflation corresponded with a transition to lower inflation variance and a *price-setting* regime (Figure 3) would be consistent with the policy trade-offs that we described in Section 5. In particular, a monetary rule that sought to induce price stability (*i.e.,* increasing α_A and leaning into productivity shocks) could have induced a regime switch from quantity-setting to price-setting (as per Propositions 7 and 8) that led the monetary contraction to have large contractionary effects on real output.

Moreover, the data suggest that the current post-Covid inflationary period favors quantity-setting. Thus, our results suggest that current monetary policy, as in the 1970s, should be especially able to control inflation with a limited trade-off of cooling output. This notwithstanding, our analysis emphasizes that policymakers need to proceed with moderation. An overly strong accommodation of productivity shocks could induce a switch to price-setting, forcing the policymaker to face the classic trade-off between price and output stabilization.

8 Conclusion

In this paper, we applied a classic mode of economic analysis—asking what choice variable is optimal—to a new context: firms' pricing and quantity-setting decisions in macroeconomic models. We showed that the nature of firms' demand curves and uncertainty itself determine whether price-setting or quantity-setting is optimal. We argue that our analysis has three important implications.

First, as a matter of pure internal consistency, the modern paradigm of studying microfounded macroeconomic models with optimizing agents and rational expectations demands that economic agents optimize over all relevant domains. The choice between price-setting and quantity-setting is one such domain and, therefore, should be optimized by agents within the model.

Second, our analysis suggests, theoretically and empirically, that macroeconomic dynamics are drastically different under price-setting and quantity-setting. In particular, in the theory and the data, money has no real effects and highly inflationary effects under quantity-setting, while money has real effects and muted inflationary effects under price-setting.

Third, monetary stabilization policy encounters new trade-offs in the presence of agents' endogenous price-setting or quantity-setting. As we show, monetary policies that stabilize output under price-setting may run the risk of switching the economy into a more volatile quantity-setting regime.

Our analysis is, however, by no means complete. Three particularly interesting avenues for future theoretical research include: studying the general-equilibrium implications of price- and quantity-setting in richer macroeconomic models; considering how this choice of choices matters for *optimal* monetary and fiscal policy; and considering richer choices of choices for firms that can do more than simply set prices or quantities (such as managing inventories or engaging in different varieties of investment). Moreover, on the empirical side, our analysis is suggestive of the importance of price- and quantity-setting as it verifies the predictions of the theory, but it does not directly measure firms' choices of choices. Directly asking firms about their pricing and production strategies and what determines them could be an important source of further tests and serve as input into positive and normative business-cycle analysis. We leave these issues to subsequent research.

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Appendices

A Omitted Derivations and Proofs

A.1 Proof of Proposition 1

Proof. We systematically work through the derivations that underlie Equation 16. We first derive the cost function c. By the first-order condition of Equation 5, we have that:

$$p_{xi} = \lambda \alpha_i \frac{q}{x_i} \tag{68}$$

where λ is the Lagrange multiplier on the constraint. Multiplying both sides by z_i and summing, we obtain that:

$$c(q; p_x, \Theta) = \sum_{i=1}^{N} p_{xi} x_i = \lambda q \tag{69}$$

Moreover, setting $q = 1$, and substituting the first-order condition into the constraint, we obtain that:

$$\lambda = \Theta^{-1} \prod_{i=1}^{I} \left(\frac{p_{xi}}{\alpha_i} \right)^{\alpha_i} \tag{70}$$

Thus, real marginal costs are given by $\mathcal{M} = P^{-1}\lambda$, as claimed in Equation 7.

To derive the optimal price, we take the first-order condition of Equation 8. This yields:

$$\eta {p^*}^{-\eta-1} \mathbb{E}\left[\Lambda \mathcal{M} P^{\eta} \Psi\right] = (\eta - 1) {p^*}^{-\eta} \mathbb{E}\left[\Lambda P^{\eta-1} \Psi\right] \tag{71}$$

which rearranges to Equation 9. Substituting p^* into Equation 8, we obtain Equation 10:

$$\begin{aligned}
V^P &= \mathbb{E}\left[\Lambda \left(\frac{p^*}{P} - \mathcal{M}\right) \Psi \left(\frac{p^*}{P}\right)^{-\eta}\right] \\
&= \mathbb{E}\left[\Lambda \left(\frac{1}{P}\frac{\eta}{\eta-1}\frac{\xi^P}{\zeta^P} - \mathcal{M}\right) \Psi P^{\eta} \left(\frac{\eta}{\eta-1}\right)^{-\eta} \left(\frac{\xi^P}{\zeta^P}\right)^{-\eta}\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta} \left[\frac{\eta}{\eta-1} \left(\frac{\xi^P}{\zeta^P}\right)^{1-\eta} \zeta^P - \left(\frac{\xi^P}{\zeta^P}\right)^{-\eta} \xi^P\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta} \left(\frac{\eta}{\eta-1} - 1\right) {\xi^P}^{1-\eta} {\zeta^P}^{\eta} \\
&= \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta} \mathbb{E}\left[\Lambda \mathcal{M} P^{\eta} \Psi\right]^{1-\eta} \mathbb{E}\left[\Lambda P^{\eta-1} \Psi\right]^{\eta}
\end{aligned} \tag{72}$$

where $\xi^P = \mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi\right]$ and $\zeta^P = \mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right]$.

To derive the optimal quantity, we take the first-order condition of Equation 11. This yields:

$$\mathbb{E}\left[\Lambda \mathcal{M}\right] = \frac{\eta - 1}{\eta} {q^*}^{-\frac{1}{\eta}} \mathbb{E}[\Lambda \Psi^{\frac{1}{\eta}}] \tag{73}$$

which rearranges to Equation 43. Substituting q^* into Equation 11, we obtain Equation 13:

$$\begin{aligned} V^Q &= \mathbb{E}\left[\Lambda\left(\left(\frac{x}{\Psi}\right)^{-\frac{1}{\eta}} - \mathcal{M}\right) q\right] \\ &= \mathbb{E}\left[\Lambda\left(\frac{\eta}{\eta-1}\frac{\xi^Q}{\zeta^Q}\Psi^{\frac{1}{\eta}} - \mathcal{M}\right)\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\xi^Q}{\zeta^Q}\right)^{-\eta}\right] \\ &= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1}\left(\frac{\xi^Q}{\zeta^Q}\right)^{1-\eta}\zeta^Q - \left(\frac{\xi^Q}{\zeta^Q}\right)^{-\eta}\xi^Q\right] \\ &= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1} - 1\right]\xi^{Q^{1-\eta}}\zeta^{Q^{\eta}} \\ &= \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\mathbb{E}\left[\Lambda\mathcal{M}\right]^{1-\eta}\mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}]^{\eta} \end{aligned} \tag{74}$$

where $\xi^Q = \mathbb{E}\left[\Lambda\mathcal{M}\right]$ and $\zeta^Q = \mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}]$.

To find Δ, we first write 15 as:

$$\Delta = \eta(\log\zeta^P - \log\zeta^Q) - (\eta - 1)(\log\xi^P - \log\xi^Q) \tag{75}$$

Thus, it suffices to compute $(\zeta^P, \zeta^Q, \xi^P, \xi^Q)$. To this end, we first prove establish that $(\Psi, P, \Lambda, \mathcal{M})$ is log-normal. We assumed that $(\Psi, P, \Theta, \Lambda, p_z)$ is log-normal. Moreover, we have that:

$$\log\mathcal{M} = -\log\Theta + \sum_{i=1}^{I}\alpha_i\log p_{zi} - \sum_{i=1}^{I}\alpha_i\log\alpha_i \tag{76}$$

which is an affine combination of jointly normal random variables, and is therefore jointly normal with (Ψ, P, Λ). Given log-normality of $(\Psi, P, \Lambda, \mathcal{M})$, we may write:

$$\begin{pmatrix}\log\Psi \\ \log P \\ \log\Lambda \\ \log\mathcal{M}\end{pmatrix} \sim N\left(\begin{pmatrix}\mu_\Psi \\ \mu_P \\ \mu_\Lambda \\ \mu_\mathcal{M}\end{pmatrix}, \begin{pmatrix}\sigma_\Psi^2 & \sigma_{\Psi,P} & \sigma_{\Psi,\Lambda} & \sigma_{\Psi,\mathcal{M}} \\ \sigma_{\Psi,P} & \sigma_P^2 & \sigma_{P,\Lambda} & \sigma_{P,\mathcal{M}} \\ \sigma_{\Psi,\Lambda} & \sigma_{P,\Lambda} & \sigma_\Lambda^2 & \sigma_{\Lambda,\mathcal{M}} \\ \sigma_{\Psi,\mathcal{M}} & \sigma_{P,\mathcal{M}} & \sigma_{\Lambda,\mathcal{M}} & \sigma_\mathcal{M}^2\end{pmatrix}\right) \tag{77}$$

□

To compute the first term, the cost-hedging cost of prices, we compute:

$$\begin{aligned}\log \xi^P &= \log \mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi\right] = \log \mathbb{E}\left[\exp\{\log \Lambda + \log \mathcal{M} + \eta \log P + \log \Psi\}\right] \\ &= \mu_\Lambda + \mu_{\mathcal{M}} + \eta \mu_P + \mu_\Psi + \frac{1}{2}\left(\sigma_\Lambda^2 + \sigma_{\mathcal{M}}^2 + \eta^2 \sigma_P^2 + \sigma_\Psi^2\right) \\ &\quad + \sigma_{\Lambda,\mathcal{M}} + \eta \sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + \eta \sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} + \eta \sigma_{P,\Psi}\end{aligned} \tag{78}$$

and

$$\begin{aligned}\log \xi^Q &= \log \mathbb{E}[\Lambda \mathcal{M}] = \log \mathbb{E}[\exp\{\log \Lambda + \log \mathcal{M}\}] \\ &= \mu_\Lambda + \mu_{\mathcal{M}} + \frac{1}{2}\left(\sigma_\Lambda^2 + \sigma_{\mathcal{M}}^2\right) + \sigma_{\Lambda,\mathcal{M}}\end{aligned} \tag{79}$$

Thus, the cost-hedging cost of prices is given by:

$$\begin{aligned}(\eta - 1)(\log \xi^P - \log \xi^Q) = (\eta - 1)\Bigg[&\eta \mu_P + \mu_\Psi + \frac{1}{2}\left(\eta^2 \sigma_P^2 + \sigma_\Psi^2\right) \\ &+ \eta \sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + \eta \sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} + \eta \sigma_{P,\Psi}\Bigg]\end{aligned} \tag{80}$$

To compute the revenue-hedging benefit of prices, we compute:

$$\begin{aligned}\log \zeta^P &= \log \mathbb{E}\left[\Lambda P^{\eta-1} \Psi\right] = \log \mathbb{E}[\exp\{\log \Lambda + (\eta - 1)\log P + \log \Psi\}] \\ &= \mu_\Lambda + (\eta - 1)\mu_P + \mu_\Psi + \frac{1}{2}\left(\sigma_\Lambda^2 + (\eta - 1)^2 \sigma_P^2 + \sigma_\Psi^2\right) \\ &\quad + (\eta - 1)\sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + (\eta - 1)\sigma_{P,\Psi}\end{aligned} \tag{81}$$

and

$$\begin{aligned}\log \zeta^Q &= \log \mathbb{E}\left[\Lambda \Psi^{\frac{1}{\eta}}\right] = \log \mathbb{E}\left[\exp\left\{\log \Lambda + \frac{1}{\eta}\log \Psi\right\}\right] \\ &= \mu_\Lambda + \frac{1}{\eta}\mu_\Psi + \frac{1}{2}\left(\sigma_\Lambda^2 + \frac{1}{\eta^2}\sigma_\Psi^2\right) + \frac{1}{\eta}\sigma_{\Lambda,\Psi}\end{aligned} \tag{82}$$

Thus, the revenue-hedging benefit of prices is given by:

$$\begin{aligned}\eta(\log \zeta^P - \log \zeta^Q) = \eta\Bigg[&(\eta - 1)\mu_P + \frac{\eta - 1}{\eta}\mu_\Psi + \frac{1}{2}\left((\eta - 1)^2 \sigma_P^2 + \left(1 - \frac{1}{\eta^2}\right)\sigma_\Psi^2\right) \\ &+ (\eta - 1)\sigma_{\Lambda,P} + \frac{\eta - 1}{\eta}\sigma_{\Lambda,\Psi} + (\eta - 1)\sigma_{P,\Psi}\Bigg]\end{aligned} \tag{83}$$

Taking the difference between the cost-hedging and revenue-hedging terms, we obtain Equation 16:

$$\begin{aligned}\Delta &= \frac{1}{2}\left[\left(\eta(\eta-1)^2 - \eta^2(\eta-1)\right)\sigma_P^2 + \left(\eta\left(1-\frac{1}{\eta^2}\right) - (\eta-1)\right)\sigma_\Psi^2\right] \\ &\quad - \eta(\eta-1)\sigma_{\mathcal{M},P} - (\eta-1)\sigma_{\mathcal{M},\Psi} \\ &= \frac{1}{2}\left(\frac{\eta-1}{\eta}\sigma_\Psi^2 - \eta(\eta-1)\sigma_P^2 - 2(\eta-1)\sigma_{\mathcal{M},\Psi} - 2\eta(\eta-1)\sigma_{\mathcal{M},P}\right)\end{aligned} \tag{84}$$

Completing the proof.

A.2 Proof of Lemma 1

Proof. We first derive Equation 30. From Equations 28 and 29, we obtain:

$$\frac{1}{M_t} + \beta\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] = \beta(1+i_t)\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] \tag{85}$$

It follows that:

$$\frac{1}{M_t} = \beta i_t\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] = \frac{i_t}{1+i_t}C_t^{-\gamma}\frac{1}{P_t} \tag{86}$$

where the second equality uses Equation 29 once again. This rearranges to Equation 30.

We next derive Equation 33. Substituting equation 30 into Equation 29, we obtain:

$$\frac{1+i_t}{i_t}\frac{1}{M_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{1}{M_{t+1}}\right] \tag{87}$$

Dividing both sides by $(1+i_t)$, multiplying by M_t, and then adding one, we obtain:

$$\frac{1+i_t}{i_t} = 1 + \beta\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{M_t}{M_{t+1}}\right] = 1 + \beta\mathbb{E}_t\left[\exp\{-\mu - \sigma_M\varepsilon_t^M\}\frac{1+i_{t+1}}{i_{t+1}}\right] \tag{88}$$

where the second equality exploits the fact that M_t follows a random walk with drift. If we guess that i_t is deterministic and define $x_t = \frac{1+i_t}{i_t}$, then we obtain that:

$$x_t = 1 + \delta_t x_{t+1} \tag{89}$$

where:

$$\delta_t = \beta\exp\{-\mu + \frac{1}{2}\sigma_{M,t}^2\} \tag{90}$$

Solving this equation forward, we obtain that:

$$x_t = 1 + \delta_t \sum_{i=1}^{T-1} \prod_{j=1}^{i} \delta_{t+j} + \delta_t \left(\prod_{j=1}^{T} \delta_{t+j} \right) x_{t+T+1} \tag{91}$$

Taking the limit $T \to \infty$, this becomes:

$$x_t = 1 + \delta_t \sum_{i=1}^{\infty} \prod_{j=1}^{i} \delta_{t+j} + \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j} \right) x_{t+T+1} \tag{92}$$

where the final term can be bounded using the fact that $\delta_t \in [0, \beta]$ (which itself follows from the assumption that $\frac{1}{2}\sigma_{M,t}^2 \leq \mu_M$):

$$0 \leq \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j} \right) x_{t+T+1} \leq \lim_{T\to\infty} \beta^{T+1} x_{t+T+1} \tag{93}$$

The household's transversality condition ensures that this upper bound is zero. Formally, the transversality condition (necessary for the optimality of the household's choices) is that:

$$\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T} (M_T + (1 + i_T)B_T) = 0 \tag{94}$$

Moreover, as $B_t = 0$ for all $t \in \mathbb{N}$, this reduces to $\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T} M_T = 0$. By Equation 86, we have that $\frac{x_t}{M_t} = \frac{C_t^{-\gamma}}{P_t}$. Thus, the transversality condition reduces to $\lim_{T\to\infty} \beta^T x_T = 0$. Combining this with Equation 93, we have that $\lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j} \right) x_{t+T+1} = 0$. Equation 33 follows:

$$\frac{1 + i_t}{i_t} = 1 + \beta \exp\{-\mu + \frac{1}{2}\sigma_{M,t}^2\} \sum_{i=1}^{\infty} \prod_{j=1}^{i} \beta \exp\{-\mu + \frac{1}{2}\sigma_{M,t+j}^2\} \tag{95}$$

The formulae in Equation 35 then follow. In particular, $\Psi_{it} = \vartheta_{it} C_t$ follows from comparing Equations 2 and 34. $P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t$ follows from equation 30. $\Lambda_t = C_t^{-\gamma}$ is the households marginal utility from consumption. Finally, $\mathcal{M}_{it} = \frac{1}{z_{it} A_t} \frac{w_{it}}{P_t} = \frac{\phi_{it} C_t^{\gamma}}{z_{it} A_t}$ follows from Equation 27.

□

A.3 Proof of Proposition 2

Proof. To work out the prices that firms set (Equation 37), we need to compute two objects $\log \mathbb{E}_{it}\left[\phi_{it}(z_{it}A_t)^{-1}P_t^{\eta}\vartheta_{it}C_t\right]$ and $\log \mathbb{E}_{it}\left[C_t^{1-\gamma}P_t^{\eta-1}\vartheta_{it}\right]$. For the first of these, we have:

$$\begin{aligned}&\log \mathbb{E}_{it}\left[\phi_{it}(z_{it}A_t)^{-1}P_t^{\eta}\vartheta_{it}C_t\right]\\&= \log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} - \log A_t + \eta \log P_t + \log \vartheta_{it} + \log C_t\right\}\right]\end{aligned} \tag{96}$$

By Lemma 1, we have that:

$$\log P_t = \log \frac{1+i^*}{i^*} + \log M_t - \gamma \log C_t \tag{97}$$

Moreover, we have conjectured that:

$$\log C_t = \chi^P_{0,t-1} + \chi^P_{A,t-1}\log A_t + \chi^P_{M,t-1}\log M_t \tag{98}$$

Thus, we can express:

$$\begin{aligned}&\log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} - \log A_t + \eta \log P_t + \log \vartheta_{it} + \log C_t\right\}\right]\\&= \eta \log \frac{1+i^*}{i^*}\\&\quad + \log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} - \log A_t + \eta \log M_t + \log \vartheta_{it} + (1-\eta\gamma)\log C_t\right\}\right]\\&= \eta \log \frac{1+i^*}{i^*} + (1-\eta\gamma)\chi^P_{0,t-1}\\&+ \log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} - \log A_t + \eta \log M_t + \log \vartheta_{it} + (1-\eta\gamma)\left(\chi^P_{A,t-1}\log A_t + \chi^P_{M,t-1}\log M_t\right)\right\}\right]\\&= \eta \log \frac{1+i^*}{i^*} + (1-\eta\gamma)\chi^P_{0,t-1}\\&+ \log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} + ((1-\eta\gamma)\chi^P_{A,t-1} - 1)\log A_t + (\eta + (1-\eta\gamma)\chi^P_{M,t-1})\log M_t + \log \vartheta_{it}\right\}\right]\end{aligned} \tag{99}$$

As ϕ_{it}, z_{it}, ϑ_{it}, A_t, and M_t are independent random variables, we can compute:

$$\begin{aligned}&\log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it} - \log z_{it} + ((1-\eta\gamma)\chi^P_{A,t-1} - 1)\log A_t + (\eta + (1-\eta\gamma)\chi^P_{M,t-1})\log M_t + \log \vartheta_{it}\right\}\right]\\&= \log \mathbb{E}_{it}\left[\exp\left\{\log \phi_{it}\right\}\right] + \log \mathbb{E}_{it}\left[\exp\left\{-\log z_{it}\right\}\right] + \log \mathbb{E}_{it}\left[\exp\left\{\log \vartheta_{it}\right\}\right]\\&\quad + \log \mathbb{E}_{it}\left[\exp\left\{((1-\eta\gamma)\chi^P_{A,t-1} - 1)\log A_t\right\}\right] + \log \mathbb{E}_{it}\left[\exp\left\{(\eta + (1-\eta\gamma)\chi^P_{M,t-1})\log M_t\right\}\right]\\&= \mu_\phi + \frac{1}{2}\sigma^2_\phi - \mu_z + \frac{1}{2}\sigma^2_z + \mu_\vartheta + \frac{1}{2}\sigma^2_\vartheta\\&+ \kappa^A_{t-1}((1-\eta\gamma)\chi^P_{A,t-1} - 1)s^A_{it} + (1-\kappa^A_{t-1})((1-\eta\gamma)\chi^P_{A,t-1} - 1)\mu^A_{t-1}\\&\quad + \frac{1}{2}((1-\eta\gamma)\chi^P_{A,t-1} - 1)^2\sigma^2_{A|s,t-1}\\&+ \kappa^M(\eta + (1-\eta\gamma)\chi^P_{M,t-1})s^M_{it} + (1-\kappa^M)(\eta + (1-\eta\gamma)\chi^P_{M,t-1})\mu^M_{t-1}\\&\quad + \frac{1}{2}(\eta + (1-\eta\gamma)\chi^P_{M,t-1})^2\sigma^2_{M|s}\end{aligned} \tag{100}$$

Thus, we have that:

$$\log \mathbb{E}_{it}\left[\phi_{it}(z_{it}A_t)^{-1}P_t^{\eta}\vartheta_{it}C_t\right] = a_{t-1} + b_{t-1}s_{it}^A + c_{t-1}s_{it}^M \tag{101}$$

$$\begin{aligned}
a_{t-1} &= \eta\log\frac{1+i^*}{i^*} + (1-\eta\gamma)\chi_{0,t-1}^P + \mu_\phi + \frac{1}{2}\sigma_\phi^2 - \mu_z + \frac{1}{2}\sigma_z^2 + \mu_\vartheta + \frac{1}{2}\sigma_\vartheta^2 \\
&\quad + (1-\kappa_{t-1}^A)((1-\eta\gamma)\chi_{A,t-1}^P - 1)\mu_{t-1}^A \\
&\quad + \frac{1}{2}((1-\eta\gamma)\chi_{A,t-1}^P - 1)^2\sigma_{A|s,t-1}^2 + (1-\kappa^M)(\eta + (1-\eta\gamma)\chi_{M,t-1}^P)\mu_{t-1}^M \\
&\quad + \frac{1}{2}(\eta + (1-\eta\gamma)\chi_{M,t-1}^P)^2\sigma_{M|s}^2 \\
b_{t-1} &= \kappa_{t-1}^A((1-\eta\gamma)\chi_{A,t-1}^P - 1) \\
c_{t-1} &= \kappa^M(\eta + (1-\eta\gamma)\chi_{M,t-1}^P)
\end{aligned} \tag{102}$$

Moving to the second conditional expectation, we have that:

$$\begin{aligned}
&\log \mathbb{E}_{it}\left[C_t^{1-\gamma}P_t^{\eta-1}\vartheta_{it}\right] \\
&= \log \mathbb{E}_{it}\left[\exp\left\{(1-\gamma)\log C_t + (\eta-1)\log P_t + \log\vartheta_{it}\right\}\right] \\
&= (\eta-1)\log\frac{1+i^*}{i^*} \\
&+ \log \mathbb{E}_{it}\left[\exp\left\{(1-\eta\gamma)\chi_{0,t-1}^P + (1-\eta\gamma)\chi_{A,t-1}^P\log A_t + \left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)\log M_t + \vartheta_{it}\right\}\right] \\
&= (\eta-1)\log\frac{1+i^*}{i^*} + (1-\eta\gamma)\chi_{0,t-1}^P + \mu_\vartheta + \frac{1}{2}\sigma_\vartheta^2 \\
&+ \kappa_{t-1}^A(1-\eta\gamma)\chi_{A,t-1}^P s_{it}^A + (1-\kappa_{t-1}^A)(1-\eta\gamma)\chi_{A,t-1}^P\mu_{t-1}^A + \frac{1}{2}(1-\eta\gamma)^2\chi_{A,t-1}^{P^2}\sigma_{A|s,t-1}^2 \\
&+ \kappa_M\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)s_{it}^M + (1-\kappa_M)\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)\mu_{t-1}^M \\
&+ \frac{1}{2}\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)^2\sigma_{M|S}^2
\end{aligned} \tag{103}$$

Thus, we have that:

$$\log \mathbb{E}_{it}\left[C_t^{1-\gamma}P_t^{\eta-1}\vartheta_{it}\right] = d_{t-1} + e_{t-1}s_{it}^A + f_{t-1}s_{it}^M \tag{104}$$

where:

$$\begin{aligned}
d_{t-1} &= (\eta-1)\log\frac{1+i^*}{i^*} + (1-\eta\gamma)\chi_{0,t-1}^P + \mu_\vartheta + \frac{1}{2}\sigma_\vartheta^2 \\
&\quad + (1-\kappa_{t-1}^A)(1-\eta\gamma)\chi_{A,t-1}^P\mu_{t-1}^A + \frac{1}{2}(1-\eta\gamma)^2\chi_{A,t-1}^{P^2}\sigma_{A|s,t-1}^2 \\
&\quad + (1-\kappa_M)\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)\mu_{t-1}^M + \frac{1}{2}\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)^2\sigma_{M|S}^2 \\
e_{t-1} &= \kappa_{t-1}^A(1-\eta\gamma)\chi_{A,t-1}^P \\
f_{t-1} &= \kappa_M\left((\eta-1) + (1-\eta\gamma)\chi_{M,t-1}^P\right)
\end{aligned} \tag{105}$$

Substituting back into the best reply, we have that:

$$\log p_{it} = \tilde{a}_{t-1} + \tilde{b}_{t-1} s_{it}^A + \tilde{c}_{t-1} s_{it}^M \tag{106}$$

where $\tilde{a}_{t-1} = \log \frac{\eta}{\eta - 1} + a_{t-1} - d_{t-1}$, $\tilde{b}_{t-1} = b_{t-1} - e_{t-1}$, $\tilde{c}_{t-1} = c_{t-1} - f_{t-1}$. Thus $\log p_{it}$ is a normal random variable. Under these normal best replies, the aggregate price level is given by the ideal price index (Equation 38):

$$\begin{aligned} \log P_t &= \frac{1}{1-\eta} \log \mathbb{E}_t \left[\exp \left\{ \log \vartheta_{it} + (1-\eta) \log p_{it} \right\} \right] \\ &= \frac{1}{1-\eta} \left(\mu_\vartheta + \frac{1}{2} \sigma_\vartheta^2 \right) + \mathbb{E}_t[\log p_{it}] + \frac{1-\eta}{2} \mathbb{V}_t[\log p_{it}] \end{aligned} \tag{107}$$

where $\mathbb{E}_t[\log p_{it}] = \tilde{a}_{t-1} + \tilde{b}_{t-1} A_t + \tilde{c}_{t-1} M_t$ and $\mathbb{V}_t[\log p_{it}] = \tilde{b}_{t-1}^2 \sigma_{s,A}^2 + \tilde{c}_{t-1}^2 \sigma_{s,M}^2$. Thus, we have that:

$$\log P_t = \frac{1}{1-\eta} \left(\mu_\vartheta + \frac{1}{2} \sigma_\vartheta^2 \right) + \tilde{a}_{t-1} + \frac{1-\eta}{2} \left(\tilde{b}_{t-1}^2 \sigma_{s,A}^2 + \tilde{c}_{t-1}^2 \sigma_{s,M}^2 \right) + \tilde{b}_{t-1} \log A_t + \tilde{c}_{t-1} \log M_t \tag{108}$$

This in turn, by Equation 30, implies that:

$$\begin{aligned} \log C_t &= -\frac{1}{\gamma} \log \frac{1+i^*}{i^*} - \frac{1}{\gamma} \log P_t + \frac{1}{\gamma} \log M_t \\ &= -\frac{1}{\gamma} \left(\log \frac{1+i^*}{i^*} + \frac{1}{1-\eta} \left(\mu_\vartheta + \frac{1}{2} \sigma_\vartheta^2 \right) + \tilde{a}_{t-1} + \frac{1-\eta}{2} \left(\tilde{b}_{t-1}^2 \sigma_{s,A}^2 + \tilde{c}_{t-1}^2 \sigma_{s,M}^2 \right) \right) \\ &\quad - \frac{1}{\gamma} \tilde{b}_{t-1} \log A_t + \frac{1}{\gamma} (1 - \tilde{c}_{t-1}) \log M_t \end{aligned} \tag{109}$$

Thus, we have a unique solution to the original conjecture, with:

$$\begin{aligned} \chi_{0,t-1}^P &= -\frac{1}{\gamma} \left(\log \frac{1+i^*}{i^*} + \frac{1}{1-\eta} \left(\mu_\vartheta + \frac{1}{2} \sigma_\vartheta^2 \right) + \tilde{a}_{t-1} + \frac{1-\eta}{2} \left(\tilde{b}_{t-1}^2 \sigma_{s,A}^2 + \tilde{c}_{t-1}^2 \sigma_{s,M}^2 \right) \right) \\ \chi_{A,t-1}^P &= -\frac{1}{\gamma} \tilde{b}_{t-1} = \frac{1}{\gamma} \kappa_{t-1}^A \\ \chi_{M,t-1}^P &= \frac{1}{\gamma} (1 - \tilde{c}_{t-1}) = \frac{1}{\gamma} (1 - \kappa_M) \end{aligned} \tag{110}$$

Completing the proof. □

A.4 Proof of Proposition 3

Proof. The overall level of consumption in the economy is given by

$$C_t = \left[\int \vartheta_{it}^{\frac{1}{\eta}} q_{it}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{111}$$

We guess that conditional on the realizations of Λ and M, the q_{it} are distributed according to a log-normal random variable. This will be confirmed under our log-linear guess for consumption later. This implies that we may write:

$$\log C_t = \mathbb{E}[\log q_{it}] + \frac{1}{2}\frac{\eta-1}{\eta} \mathrm{Var} \left(\log q_{it} + \left(\frac{\eta}{\eta-1} \right)^2 \frac{1}{\eta^2} \log \vartheta_{it} \right) \tag{112}$$

where the expectation and variance are over the cross-sectional distribution of the q_{it}'s. Recall that $\log q_{it}$ is given by:

$$\log q_{it} = -\eta \left[\log \left(\frac{\eta}{\eta-1} \right) + \log \mathbb{E}_{it} \left[\phi_{it}(z_{it}A_t)^{-1} \right] - \mathbb{E}_{it} \left[\vartheta_{it}^{\frac{1}{\eta}} C_t^{-\gamma+\frac{1}{\eta}} \right] \right] \tag{113}$$

We guess that

$$\log C_t = \chi_{0,t-1}^Q + \chi_{A,t-1}^Q \log A_t + \chi_{M,t-1}^Q \log M_t \tag{114}$$

We proceed by substituting our guess into (113). To ease notation, define δ_x, ζ_x as the *precision* of the prior and the signal at time t corresponding to variable x, respectively. Further, let μ_x denote the prior mean of variable x at time t. We simplify (113) term by term to obtain the following:

$$\begin{aligned}
\log \mathbb{E}_{it} \left[\phi_{it}(z_{it}A_t)^{-1} \right] =& - \mu_z + \mu_\phi + \frac{1}{2}(\delta_z)^{-1} + \frac{1}{2}(\delta_\phi)^{-1} - \left(\frac{\zeta_A}{\zeta_A + \delta_A} s_{it}^A + \frac{\delta_A}{\zeta_A + \delta_A} \mu_A \right) + \frac{1}{2}(\zeta_A + \delta_A)^{-1} \\
\log \mathbb{E}_{it} \left[\vartheta^{\frac{1}{\eta}} C^{-\gamma+\frac{1}{\eta}} \right] =& \left(-\gamma + \frac{1}{\eta} \right) \chi_{0,t-1}^Q + \chi_{A,t-1}^Q \left(-\gamma + \frac{1}{\eta} \right) \left(\frac{\zeta_A}{\zeta_A + \delta_A} s_{it}^A + \frac{\delta_A}{\zeta_A + \delta_\Lambda} \mu_A \right) \\
&+ \frac{1}{2} \left(\chi_{A,t-1}^Q \right)^2 \left(-\gamma + \frac{1}{\eta} \right)^2 (\zeta_A + \delta_A)^{-1} + \frac{1}{\eta} \mu_\vartheta + \frac{1}{2\eta^2} (\delta_\vartheta)^{-1} \\
&+ \chi_{M,t-1}^Q \left(-\gamma + \frac{1}{\eta} \right) \left(\frac{\zeta_M}{\zeta_M + \delta_M} s_{it}^M + \frac{\delta_M}{\zeta_M + \delta_M} \mu_M \right) \\
&+ \frac{1}{2} \left(\chi_{M,t-1}^Q \right)^2 \left(-\gamma + \frac{1}{\eta} \right)^2 (\zeta_M + \delta_M)^{-1}
\end{aligned} \tag{115}$$

we can now collect terms after observing that $\mathbb{E}[s_{it}^A] = A_t$ and $\mathbb{E}[s_{it}^M] = M_t$ where the expectation is again over i. Collecting all terms for A_t and equating coefficients yields:

$$\chi_{A,t-1}^Q = \eta \left[\left(\frac{\zeta_A}{\zeta_A + \delta_A} \right) + \left(\frac{1}{\eta} - \gamma \right) \left(\frac{\zeta_A}{\zeta_A + \delta_A} \right) \chi_{A,t-1}^Q \right] \tag{116}$$

for which we obtain

$$\chi^Q_{A,t-1} = \frac{1}{\frac{\zeta_A+\delta_A}{\zeta_A}\left(\frac{1}{\eta}\right) - \frac{1}{\eta} + \gamma} \tag{117}$$

which is exactly the equation in the main text after substituting $\delta_A = \sigma^{-2}_{A,t-1}$ and $\zeta_A = \left(\sigma^A_s\right)^{-2}$. Similarly, we may collect all terms on M_t to obtain:

$$\chi^Q_{M,t-1} = \eta\left(\frac{1}{\eta} - \gamma\right)\left(\frac{\zeta_A}{\zeta_A+\delta_A}\right)\chi^Q_{M,t-1} \tag{118}$$

for which we obtain $\chi^Q_{M,t-1} = 0$. Note further that all other terms only depend on $t-1$ through δ_A. This verifies our conjecture that consumption is log-normal in aggregates as well as our conjecture that quantities are distributed log-normally across firms (conditional on A_t and M_t). In order to obtain the expression for the price level, note that (35) implies that

$$\log P_t = \log\frac{i^*}{1+i^*} - \gamma\log C_t + \log M_t \tag{119}$$

And therefore we have

$$\log P_t = \gamma(\log\kappa - \log\chi_{0,t-1}) - \gamma\chi^Q_{A,t-1}\log A_t + \left(1 - \gamma\chi^Q_{M,t-1}\right)\log M_t \tag{120}$$

The result follows. □

A.5 Proof of Lemma 2

Proof. From Proposition 1, we have that:

$$\Delta = \frac{1}{2}(\eta-1)\left(\frac{1}{\eta}\sigma^2_\Psi - \eta\sigma^2_P - 2\sigma_{\Psi,\mathcal{M}} - 2\eta\sigma_{P,\mathcal{M}}\right) \tag{121}$$

Moreover, applying Lemma 1, we have that:

$$\sigma^2_\Psi = \sigma^2_\vartheta + \sigma^2_C \tag{122}$$

$$\sigma^2_P = \gamma^2\sigma^2_C + \sigma^2_M - 2\gamma\sigma_{C,M} \tag{123}$$

$$\sigma_{\Psi,\mathcal{M}} = \gamma\sigma^2_C - \sigma_{C,A} \tag{124}$$

$$\sigma_{P,\mathcal{M}} = \gamma\sigma_{C,A} - \gamma^2\sigma^2_C + \gamma\sigma_{C,M} \tag{125}$$

Substituting these formulae, we obtain Equation 49:

$$\Delta = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\vartheta}^{2} + \frac{1}{\eta}\left(1-\eta\gamma\right)^{2}\sigma_{C}^{2} - \eta\sigma_{M}^{2} + 2(1-\eta\gamma)\sigma_{C,A}\right) \tag{126}$$

Moreover, applying Propositions 2 and 3, we have that these variances for the firm in each of the price-setting and quantity-setting regimes are given by:

$$\sigma_{C,t-1}^{2P} = \chi_{A,t-1}^{P2}\sigma_{A|s,t-1}^{2} + \chi_{M}^{P2}\sigma_{M|s}^{2} \tag{127}$$

$$\sigma_{C,A,t-1}^{P} = \chi_{A,t-1}^{P}\sigma_{A|s,t-1}^{2} \tag{128}$$

$$\sigma_{C,t-1}^{2Q} = \chi_{A,t-1}^{2Q}\sigma_{A|s,t-1}^{2} \tag{129}$$

$$\sigma_{C,A,t-1}^{Q} = \chi_{A,t-1}^{Q}\sigma_{A|s,t-1}^{2} \tag{130}$$

Thus, if all other firms set quantities:

$$\Delta_{t}^{Q} = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\vartheta}^{2} - \eta\sigma_{M|s}^{2} + \left(\frac{1}{\eta}(1-\eta\gamma)\chi_{A,t-1}^{Q} + 2\right)(1-\eta\gamma)\chi_{A,t-1}^{Q}\sigma_{A|s,t-1}^{2}\right) \tag{131}$$

And if all other firms set prices:

$$\begin{aligned}\Delta_{t}^{P} = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta}^{2} + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^{2}\chi_{M}^{P^{2}}\right)\sigma_{M|s}^{2} \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\chi_{A,t-1}^{P} + 2\right)(1-\eta\gamma)\chi_{A,t-1}^{P}\sigma_{A|s,t-1}^{2}\Bigg)\end{aligned} \tag{132}$$

Substituting coefficients and exploiting the fact that the conditional variances are given by $\sigma_{A|s,t-1}^{2} = \kappa_{t-1}^{A}\sigma_{A,s}^{2}$ and $\sigma_{M|s}^{2} = \kappa^{M}\sigma_{M,s}^{2}$, we obtain:

$$\begin{aligned}\Delta_{t}^{Q} = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta}^{2} - \eta\kappa_{m}\sigma_{M,s}^{2} \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa_{t-1}^{A}}{1-\kappa_{t-1}^{A}(1-\eta\gamma)} + 2\right)(1-\eta\gamma)\frac{\eta\kappa_{t-1}^{A}}{1-\kappa_{t-1}^{A}(1-\eta\gamma)}\kappa_{t-1}^{A}\sigma_{A,s}^{2}\Bigg)\end{aligned} \tag{133}$$

$$\begin{aligned}\Delta_{t}^{P} = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta}^{2} + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^{2}\left(\frac{1-\kappa^{M}}{\gamma}\right)^{2}\right)\kappa^{M}\sigma_{M,s}^{2} \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_{t-1}^{A}}{\gamma} + 2\right)(1-\eta\gamma)\frac{\kappa_{t-1}^{A}}{\gamma}\kappa_{t-1}^{A}\sigma_{A,s}^{2}\Bigg)\end{aligned} \tag{134}$$

yielding the claimed expressions. $\square$

A.6 Proof of Proposition 4

Proof. Define $\Delta\Delta_t = \Delta_t^P - \Delta_t^Q$ and observe that:

$$\Delta\Delta_t = \frac{1}{2}(\eta - 1)\Bigg[\frac{1}{\eta}(1-\eta\gamma)^2\left(\frac{1-\kappa^M}{\gamma}\right)^2\kappa^M\sigma_{M,s}^2 \\ + \left(\frac{1}{\eta}(1-\eta\gamma)^2\left(\chi_{A,t-1}^{P^2} - \chi_{A,t-1}^{Q^2}\right) + 2(1-\eta\gamma)(\chi_{A,t-1}^P - \chi_{A,t-1}^Q)\right)\kappa_{t-1}^A\sigma_{A,s}^2\Bigg] \tag{135}$$

First, when $\eta\gamma = 1$, we have that $\Delta\Delta_t = 0$. Second, suppose that $\eta\gamma < 1$. We observe that the first term in brackets is strictly positive. Turning to the second term, as $\eta\gamma < 1$, we have that $\Delta\Delta_t > 0$ if and only if $\chi_{A,t-1}^P > \chi_{A,t-1}^Q$. This inequality is equivalent to:

$$\frac{\kappa_{t-1}^A}{\gamma} > \frac{\eta\kappa_{t-1}^A}{1-\kappa_{t-1}^A(1-\eta\gamma)} \tag{136}$$

As $\eta\gamma < 1$ and $\kappa_{t-1}^A \in (0,1)$, we have that the denominator on the right-hand side is positive. Thus, we can re-express this required inequality as:

$$1 - \eta\gamma > \kappa_{t-1}^A(1-\eta\gamma) \tag{137}$$

which is true as $\eta\gamma < 1$ and $\kappa_{t-1}^A \in (0,1)$. Thus, $\Delta\Delta_t > 0$ when $\eta\gamma < 1$. Third, suppose that $\eta\gamma > 1$. Once again, the first term in brackets is strictly positive. Thus, it suffices to show that:

$$\frac{1}{\eta}(1-\eta\gamma)^2\left(\chi_{A,t-1}^{P^2} - \chi_{A,t-1}^{Q^2}\right) + 2(1-\eta\gamma)(\chi_{A,t-1}^P - \chi_{A,t-1}^Q) > 0 \tag{138}$$

See that we can factor the left-hand side of this expression as:

$$(1-\eta\gamma)(\chi_{A,t-1}^P - \chi_{A,t-1}^Q)\left(\frac{1}{\eta}(1-\eta\gamma)(\chi_{A,t-1}^P + \chi_{A,t-1}^Q) + 2\right) \tag{139}$$

By the reverse of the logic in part two, we have that $\chi_{A,t-1}^P < \chi_{A,t-1}^Q$. Thus, the expression in question is strictly positive if and only if:

$$2 > \frac{1}{\eta}(\eta\gamma - 1)(\chi_{A,t-1}^P + \chi_{A,t-1}^Q) \tag{140}$$

We now observe that $\chi^P_{A,t-1} + \chi^Q_{A,t-1} < 2\chi^Q_{A,t-1}$. Moreover, $\chi^Q_{A,t-1}$ is an increasing function of κ^A_{t-1} and is therefore bounded above by $\frac{\eta}{1+\eta\gamma-1} = \frac{1}{\gamma}$. Thus, we have that:

$$\frac{1}{\eta}(\eta\gamma - 1)(\chi^P_{A,t-1} + \chi^Q_{A,t-1}) < \frac{2}{\eta\gamma}(\eta\gamma - 1) = 2 - \frac{2}{\eta\gamma} < 2 \tag{141}$$

This establishes that $\Delta\Delta_t > 0$ if $\eta\gamma > 1$. Taken together, we have shown that $\Delta\Delta_t \geq 0$ and $\Delta\Delta_t > 0$ if and only if $\eta\gamma \neq 1$, establishing the claim. □

A.7 Proof of Corollary 3

Proof. We have shown that $\Delta^P_t \geq \Delta^Q_t$. There are two possible cases. First, suppose that $\Delta^Q_t \geq 0$. In this case, $\Delta^P_t \geq 0$ and there exists a price-setting equilibrium. Second, suppose that $\Delta^Q_t < 0$. In this case, a quantity-setting equilibrium exists. Thus, there always exists a price-setting equilibrium and/or a quantity-setting equilibrium. □

A.8 Proof of Proposition 5

Proof. We proof the claims in each case.

1. $\eta\gamma = 1$. By Lemma 2, we have that $\Delta^Q_t = \Delta^Q(0)$ and $\Delta^P_t = \Delta^Q(0)$, which are both independent of κ^A_{t-1}.
2. $\eta\gamma < 1$. By Lemma 2, we have that Δ^Q_t is strictly increasing in κ^A_{t-1} if and only if

$$\left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa^A_{t-1}}{1-\kappa^A_{t-1}(1-\eta\gamma)} + 2\right)(1-\eta\gamma)\frac{\eta\left(\kappa^A_{t-1}\right)^2}{1-\kappa^A_{t-1}(1-\eta\gamma)} \tag{142}$$

   is strictly increasing in κ^M_{t-1}. As $\eta\gamma < 1$ and $\frac{\eta\kappa^A_{t-1}}{1-\kappa^A_{t-1}(1-\eta\gamma)}$ is strictly increasing in κ^A_{t-1} and strictly positive, we have that the term in parentheses is strictly positive and strictly increasing. The term outside parentheses is strictly increasing and strictly positive for the same reasons. Moreover, Δ^P_t is strictly increasing in κ^A_{t-1} if and only if:

$$\left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa^A_{t-1}}{\gamma} + 2\right)(1-\eta\gamma)\frac{\left(\kappa^A_{t-1}\right)^2}{\gamma} \tag{143}$$

   is strictly increasing in κ^A_{t-1}. As $\eta\gamma < 1$, this is immediate.
   As Δ^Q_t and Δ^P_t are strictly increasing, they either have a unique interior root or no interior roots. Δ^Q_t has a unique interior root if and only if $\Delta^Q(0) < 0$ and $\Delta^Q(1) > 1$. In this case, let $\bar{\kappa}^A_Q \in (0,1)$ be the unique interior root. If $\Delta^Q(0) \geq 0$, then let $\bar{\kappa}^A_Q = 0$, and if $\Delta^Q(1) \leq 0$, then let $\bar{\kappa}^A_Q = 1$. Δ^P_t has a unique interior root if and only if

$\Delta^P(0) < 0$ and $\Delta^P(1) > 1$. In this case, let $\bar{\kappa}^A_P \in (0,1)$ be the unique interior root. If $\Delta^P(0) \geq 0$, then let $\bar{\kappa}^A_P = 0$, and if $\Delta^P(1) \leq 0$, then let $\bar{\kappa}^A_P = 1$. By Proposition 4, we have that $\Delta^P_t \geq \Delta^Q_t$, which establishes that $\bar{\kappa}^A_Q \leq \bar{\kappa}^A_P$.

3. $\eta\gamma > 1$. By Lemma 2, we have that Δ^Q_t is strictly decreasing in κ^A_{t-1} if and only if Expression 142 is strictly decreasing in κ^A_{t-1}. Define $\omega = 1 - \eta\gamma$ and observe that we need to show that:

$$\left(\frac{\omega\kappa^A_{t-1}}{1-\omega\kappa^A_{t-1}} + 2\right)\frac{\omega\left(\kappa^A_{t-1}\right)^2}{1-\omega\kappa^A_{t-1}} \tag{144}$$

is a strictly decreasing function of κ^A_{t-1}. Taking the derivative of this expression and rearranging, we require that:

$$\omega\kappa^A_{t-1}\left(\omega^2\left(\kappa^A_{t-1}\right)^2 - 3\omega\kappa^A_{t-1} + 4\right) < 0 \tag{145}$$

As $\omega < 0$, we require that $\omega^2\left(\kappa^A_{t-1}\right)^2 - 3\omega\kappa^A_{t-1} + 4 > 0$. This is positive if the quadratic on the left-hand side has no real roots. As $9\omega^2 - 16\omega^2 < 0$, the quadratic indeed has no real roots and so Δ^Q_t is strictly decreasing in κ^A_{t-1}.

Δ^P_t is strictly decreasing in κ^A_{t-1} if and only:

$$\left(\frac{\omega}{1-\omega}\kappa^A_{t-1} + 2\right)\frac{\omega}{1-\omega}\left(\kappa^A_{t-1}\right)^2 \tag{146}$$

is strictly decreasing in κ^A_{t-1}. Taking the derivative of this expression and rearranging, we require that:

$$\kappa^A_{t-1} < \frac{4}{3}\frac{\omega-1}{\omega} \tag{147}$$

which is always satisfied as $\omega < 0$. As Δ^Q_t and Δ^P_t are strictly decreasing, the same steps as the strictly increasing case establish the result.

□

A.9 Proof of Proposition 6

Proof. We first begin by showing that the interest rate is constant under this new monetary rule. Note that the logarithm of money in first differences is a normal random variable with mean

$$\mu_M + \alpha_A\mu_A \tag{148}$$

and variance

$$\alpha_A^2\sigma_A^2 + \sigma_m^2 \tag{149}$$

We can therefore apply the analysis in 4.1 to obtain:

$$1 + i^* = \beta^{-1} \exp \left\{ \mu_M + \alpha_A \mu_A - \frac{1}{2} \left(\alpha_A^2 \sigma_A^2 + \sigma_m^2 \right) \right\} \tag{150}$$

Noting that the interest rate is independent of A_t and m_t, we guess that prices across firms are log-normally distributed across i. This implies that we may write

$$\log P_t = \mathbb{E}[\log p_{it}] + \frac{1-\eta}{2} \mathrm{Var} \left(\log p_{it} + \frac{1}{(1-\eta)^2} \sigma_\vartheta^2 \right) \tag{151}$$

We also guess that the aggregate price level is log-linear in productivity and the money shock:

$$\log P_t = \log \chi_{0,t-1} + \chi_1 \log A_t + \chi_2 \log m_t \tag{152}$$

Recall that p_{it} is given by

$$\log p_{it} = \log \frac{\eta}{\eta - 1} + \log \mathbb{E}_{it}[\phi_{it}(z_{it}A_t)^{-1} P^\eta C \vartheta_{it}] - \log \mathbb{E}_{it}[C^{1-\gamma} P^{\eta-1} \vartheta_{it}] \tag{153}$$

We simplify each term individually. We first make use of the relationship between consumption and real money balances derived in Lemma 1, the equation for the monetary policy rule (52), and the law of motion for the aggregate price level (152) to simplify the first expectation term:

$$\mathbb{E}_{it} \left[\phi_{it}(z_{it}A_t)^{-1} P^\eta C \vartheta_{it} \right] = \mathbb{E}_{it} \left[\phi_{it}(z_{it})^{-1} \chi_0^{\eta - \frac{1}{\gamma}} A^{\chi_1\left(\eta - \frac{1}{\gamma}\right) + \frac{\alpha_A}{\gamma} - 1} m^{\chi_2\left(\eta - \frac{1}{\gamma}\right) + \frac{\sigma_m}{\gamma}} \vartheta_{it} \left(\frac{i}{1+i} \right)^{\frac{1}{\gamma}} \right] \tag{154}$$

Simplifying this expression yields a constant independent of i and two terms involving i-specific signals. We list them separately. The constant is given by:

$$\begin{aligned} &\left(\eta - \frac{1}{\gamma}\right) \log \chi_0 + \mu_\phi - \mu_z + \mu_\vartheta + \frac{1}{2}\left(\sigma_\phi^2 + \sigma_z^2 + \sigma_\vartheta^2\right) + \frac{1}{\gamma} \log \frac{i}{1+i} + \frac{1}{\gamma}\mu_M + \frac{1}{\gamma} \log M_{t-1} \\ &+ \left[\chi_1 \left(\eta - \frac{1}{\gamma}\right) + \frac{\alpha_A}{\gamma} - 1\right] \frac{\sigma_A^{-2}}{\sigma_A^{-2} + \sigma_{A,s}^{-2}} \mu_A + \frac{1}{2}\left[\chi_1 \left(\eta - \frac{1}{\gamma}\right) + \frac{\alpha_A}{\gamma} - 1\right]^2 \frac{\sigma_A^2 \sigma_{A,s}^2}{\sigma_A^2 + \sigma_{A,s}^2} \\ &+ \left[\chi_2 \left(\eta - \frac{1}{\gamma}\right) + \frac{\sigma_m}{\gamma}\right] \frac{1}{1 + \sigma_{m,s}^{-2}} \mu_m + \frac{1}{2}\left[\chi_2 \left(\eta - \frac{1}{\gamma}\right) + \frac{\sigma_m}{\gamma}\right]^2 \frac{\sigma_{m,s}^2}{1 + \sigma_{m,s}^2} \end{aligned} \tag{155}$$

The i-specific terms are given by:

$$\left[\chi_1\left(\eta-\frac{1}{\gamma}\right)+\frac{\alpha_A}{\gamma}-1\right]\frac{\sigma_{A,s}^{-2}}{\sigma_A^{-2}+\sigma_{A,s}^{-2}}s_{it}^A+\left[\chi_2\left(\eta-\frac{1}{\gamma}\right)+\frac{\sigma_m}{\gamma}\right]\frac{\sigma_{m,s}^{-2}}{1+\sigma_{m,s}^{-2}}s_{it}^m \tag{156}$$

We may simplify the second expectation term in a similar fashion and collect the constants and i-independent terms separately. We begin with the constant:

$$\begin{aligned}\left(\eta-\frac{1}{\gamma}\right)\log\chi_0++\mu_\vartheta+\frac{1}{2}\sigma_\vartheta^2+\left(\frac{1}{\gamma}-1\right)\frac{i}{1+i}+\left(\frac{1}{\gamma}-1\right)\mu_M+\left(\frac{1}{\gamma}-1\right)\log M_{t-1}\\+\left[(\alpha_A-\chi_1)\left(\frac{1}{\gamma}-1\right)+\chi_1(\eta-1)\right]\frac{\sigma_A^{-2}}{\sigma_A^{-2}+\sigma_{A,s}^{-2}}\mu_A+\frac{1}{2}\left[(\alpha_A-\chi_1)\left(\frac{1}{\gamma}-1\right)+\chi_1(\eta-1)\right]^2\frac{\sigma_A^2\sigma_{A,s}^2}{\sigma_A^2+\sigma_{A,s}^2}\\+\left[(\sigma-\chi_2)\left(\frac{1}{\gamma}-1\right)+\chi_2(\eta-1)\right]\frac{1}{1+\sigma_{m,s}^{-2}}\mu_m+\frac{1}{2}\left[(\sigma-\chi_2)\left(\frac{1}{\gamma}-1\right)+\chi_2(\eta-1)\right]^2\frac{\sigma_{m,s}^2}{1+\sigma_{m,s}^2}\end{aligned} \tag{157}$$

The i-specific terms are given by

$$\left[(\alpha_A-\chi_1)\left(\frac{1}{\gamma}-1\right)+\chi_1(\eta-1)\right]\frac{\sigma_{A,s}^{-2}}{\sigma_A^{-2}+\sigma_{A,s}^{-2}}s_{it}^A+\left[(\sigma-\chi_2)\left(\frac{1}{\gamma}-1\right)+\chi_2(\eta-1)\right]\frac{\sigma_{m,s}^{-2}}{1+\sigma_{m,s}^{-2}}s_{it}^m \tag{158}$$

Collecting all terms with s_{it}^A from both expressions, noting that $\mathbb{E}[s_{it}^A]=A_t$ (where the expectation is over i), and equating coefficients with (152) yields the following equation for χ_1:

$$\chi_1=\left[\chi_1\left(\eta-\frac{1}{\gamma}\right)+\frac{\alpha_A}{\gamma}-1\right]\frac{\sigma_{A,s}^{-2}}{\sigma_A^{-2}+\sigma_{A,s}^{-2}}-\left[(\alpha_A-\chi_1)\left(\frac{1}{\gamma}-1\right)+\chi_1(\eta-1)\right]\frac{\sigma_{A,s}^{-2}}{\sigma_A^{-2}+\sigma_{A,s}^{-2}} \tag{159}$$

which we may solve to obtain

$$\chi_1=\frac{\alpha_A-1}{1+\frac{\sigma_{A,s}^2}{\sigma_A^2}}=(\alpha_A-1)\kappa^A \tag{160}$$

Repeating the steps above yields the following equation for χ_2

$$\chi_2=\left[\chi_2\left(\eta-\frac{1}{\gamma}\right)+\frac{\sigma_m}{\gamma}\right]\frac{\sigma_{m,s}^{-2}}{1+\sigma_{m,s}^{-2}}-\left[(\sigma-\chi_2)\left(\frac{1}{\gamma}-1\right)+\chi_2(\eta-1)\right]\frac{\sigma_{m,s}^{-2}}{1+\sigma_{m,s}^{-2}} \tag{161}$$

which we may solve to obtain

$$\chi_2 = \sigma_m \kappa_m \tag{162}$$

By Lemma 1, the coefficient on A_t for consumption is given by

$$\frac{1}{\gamma}(\alpha_A - \chi_1) = \frac{1}{\gamma}\left(\alpha_A(1-\kappa_A) + \kappa_A\right)) \tag{163}$$

while the coefficient on the monetary shock is given by

$$\frac{1}{\gamma}\sigma_m(1-\kappa_m) \tag{164}$$

Finally, in order to obtain the coefficients on output, we can proceed exactly as in Appendix A.4, where we instead assume that consumption is log-linear in the monetary shock m_t instead of M_t. Because m_t does not appear in (43), the coefficient $\chi^Q_{M,t-1}$ continues to be zero, while the coefficient $\chi^Q_{A,t-1}$ remains the same. The coefficients on the evolution for the aggregate price level can then be found using Lemma 1. The proof follows. □

A.10 Proof of Proposition 7

Proof. From Proposition 1, we have that:

$$\Delta = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma^2_\Psi - \eta\sigma^2_P - 2\sigma_{\Psi,\mathcal{M}} - 2\eta\sigma_{P,\mathcal{M}}\right) \tag{165}$$

Moreover, applying Lemma 1, we have that:

$$\sigma^2_\Psi = \sigma^2_\vartheta + \sigma^2_C \tag{166}$$

$$\sigma^2_P = \gamma^2\sigma^2_C + \sigma^2_M - 2\gamma\sigma_{C,M} \tag{167}$$

$$\sigma_{\Psi,\mathcal{M}} = \gamma\sigma^2_C - \sigma_{C,A} \tag{168}$$

$$\sigma_{P,\mathcal{M}} = \gamma\sigma_{C,A} - \gamma^2\sigma^2_C + \gamma\sigma_{C,M} - \sigma_{A,M} \tag{169}$$

Substituting these formulae, we obtain:

$$\Delta = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma^2_\vartheta + \frac{1}{\eta}\left(1-\eta\gamma\right)^2\sigma^2_C - \eta\sigma^2_M + 2(1-\eta\gamma)\sigma_{C,A} + 2\eta\sigma_{A,M}\right) \tag{170}$$

Now $\sigma_{A,M} = \alpha_A \kappa^A \sigma^2_{A,s}$ by assumption and $\sigma^2_M = \alpha^2_A \kappa^A \sigma^2_{A,s} + \kappa^m \sigma^2_{m,s}$. Thus, we have that:

$$\Delta = \frac{1}{2}(\eta-1)\left(\frac{1}{\eta}\sigma^2_\vartheta + \frac{1}{\eta}(1-\eta\gamma)^2\sigma^2_C - \eta\left(\alpha^2_A\kappa^A\sigma^2_{A,s} - 2\alpha_A\kappa^A\sigma^2_{A,s} + \kappa^m\sigma^2_{m,s}\right) + 2(1-\eta\gamma)\sigma_{C,A}\right) \tag{171}$$

By Proposition 6, σ^2_C and $\sigma_{C,A}$ do not depend on α_A. Thus,

$$\frac{\partial \Delta^Q}{\partial \alpha_A} = -\eta(\eta-1)\kappa^A\sigma^2_{A,s}(\alpha_A - 1) \tag{172}$$

which is strictly positive when $\alpha_A < 1$ and strictly negative when $\alpha_A > 1$. Moreover, $\frac{\partial^2\Delta^Q}{\partial\alpha^2_A} = -\eta(\eta-1)\kappa^A\sigma^2_{A,s} < 0$, so Δ^A is strictly concave in α_A. Moreover, inspecting the formula, $\Delta^Q(\alpha_A) > \Delta^Q(0)$ if and only if $\alpha_A \in (0, 2)$. □

A.11 Proof of Proposition 8

Proof. By Proposition 7, we have that:

$$\Delta^P = \frac{1}{2}(\eta-1)\left(\frac{1}{\eta}\sigma^2_\vartheta + \frac{1}{\eta}(1-\eta\gamma)^2\sigma^2_C - \eta\left(\alpha^2_A\kappa^A\sigma^2_{A,s} - 2\alpha_A\kappa^A\sigma^2_{A,s} + \kappa^m\sigma^2_{m,s}\right) + 2(1-\eta\gamma)\sigma_{C,A}\right) \tag{173}$$

Moreover, by Proposition 6, in a price-setting regime we have that:

$$\sigma^2_C = \frac{\left(\alpha_A(1-\kappa^A)+\kappa^A\right)^2}{\gamma^2}\kappa^A\sigma^2_{A,s} + \frac{(1-\kappa^m)^2}{\gamma^2}\kappa^m\sigma^2_{m,s} \tag{174}$$

$$\sigma_{C,A} = \frac{\alpha_A(1-\kappa^A)+\kappa^A}{\gamma}\kappa^A\sigma^2_{A,s} \tag{175}$$

We now need to determine the behavior of $\Delta^P(\alpha)$. Combining Equations 173 and 174, we obtain:

$$\begin{aligned}\Delta^P(\alpha_A) = cons. + \frac{1}{2}\eta(\eta-1)\kappa^A\sigma^2_{A,s}\Bigg(&\left[\left(\frac{1-\eta\gamma}{\gamma\eta}\right)^2(1-\kappa_A)^2 - 1\right]\alpha^2_A \\ &+ 2\left[\left(\frac{1-\eta\gamma}{\eta\gamma}\right)^2(1-\kappa_A)\kappa_A + \frac{1-\eta\gamma}{\eta\gamma}(1-\kappa_A) + 1\right]\alpha_A\Bigg)\end{aligned} \tag{176}$$

where *cons.* is independent of α_A.

The linear part of Δ^P is increasing when:

$$\left(\frac{1-\eta\gamma}{\eta\gamma}\right)^2(1-\kappa_A)\kappa_A + \frac{1-\eta\gamma}{\eta\gamma}(1-\kappa_A)+1>0 \tag{177}$$

When $\eta\gamma \leq 1$, all terms on the left-hand side are positive and the inequality holds. When $\eta\gamma > 1$, we require that:

$$\left(\frac{\eta\gamma-1}{\eta\gamma}\right)^2(1-\kappa_A)\kappa_A + 1 > \frac{\eta\gamma-1}{\eta\gamma}(1-\kappa_A) \tag{178}$$

The first term on the left-hand side is strictly positive. Moreover, the term on the right-hand side is strictly less than one as $\frac{\eta\gamma-1}{\eta\gamma}\in(0,1)$ (because $\eta\gamma>1$) and $\kappa^A\in(0,1)$. Hence, $\Delta^{P'}(0)>0$, as we have claimed.

Moreover, Δ^P is a concave function whenever:

$$\left(\frac{1-\eta\gamma}{\gamma\eta}\right)^2(1-\kappa_A)^2<1 \tag{179}$$

When $\eta\gamma\geq 1$, this is always satisfied as $\kappa^A\in(0,1)$. When $\eta\gamma<1$, this is satisfied whenever: $\eta\gamma>\frac{1-\kappa^A}{2-\kappa^A}\in(0,\frac{1}{2})$. Moreover, when Δ^P is concave, we have that Δ^P is increasing up until α^*, where α^* solves:

$$\alpha^*\left[\left(\frac{1-\eta\gamma}{\gamma\eta}\right)^2(1-\kappa_A)^2-1\right]+\left[\left(\frac{1-\eta\gamma}{\eta\gamma}\right)^2(1-\kappa_A)\kappa_A+\frac{1-\eta\gamma}{\eta\gamma}(1-\kappa_A)+1\right]=0 \tag{180}$$

Rearranging yields Equation 58. In the convex case, Δ^P is increasing after α^* and decreasing before α^*. □

A.12 Proof of Proposition 9

Proof. Suppose that

$$\log C_t = \chi_0 + \chi_1 \log A_t + \chi_2 \log m_t \tag{181}$$

We may use Equation (171) to express Δ as a function of χ_1 and χ_2:

$$\begin{aligned}\Delta =&\frac{1}{2}(\eta-1)\left(\frac{1}{2}\sigma_\vartheta^2+\left(\frac{1}{\eta}(1-\eta\gamma)^2\chi_2^2-\eta\right)\kappa^m\sigma_{m,s}^2+\left(\frac{1}{\eta}(1-\eta\gamma)\chi_1+2\right)(1-\eta\gamma)\chi_1\kappa^A\sigma_{A,s}^2\right)\\ &+\frac{1}{2}(\eta-1)\left((2-\alpha_A)\alpha_A\eta\kappa_A\sigma_{A,s}^2\right)\end{aligned}$$

We can therefore right $\Delta^P - \Delta^Q$ as

$$\frac{1}{2}(\eta-1)\left(\frac{1}{\eta}(1-\eta\gamma)^2(\chi_2^P)^2\kappa^m\sigma_{m,s}^2+\frac{1}{\eta}(1-\eta\gamma)^2\left[(\chi_1^P)^2-(\chi_1^Q)^2\right]\kappa^A\sigma_{A,s}^2+2(1-\eta\gamma)(\chi_1^P-\chi_1^Q)\kappa_A\sigma_{A,s}^2\right)$$

where χ_j^P and χ_j^Q, $j \in \{1,2\}$ denote the dynamics of the economy in Proposition 6, under price-setting and quantity-setting, respectively. We now derive sufficient conditions for $\Delta^P - \Delta^Q \geq 0$. Note that this is always true if $\eta\gamma < 1$ and $\chi_1^P \geq \chi_1^Q$. This is true if and only if

$$\alpha_A \geq \frac{\kappa^A(\eta\gamma-1)}{1-\kappa^A(1-\eta\gamma)} \tag{182}$$

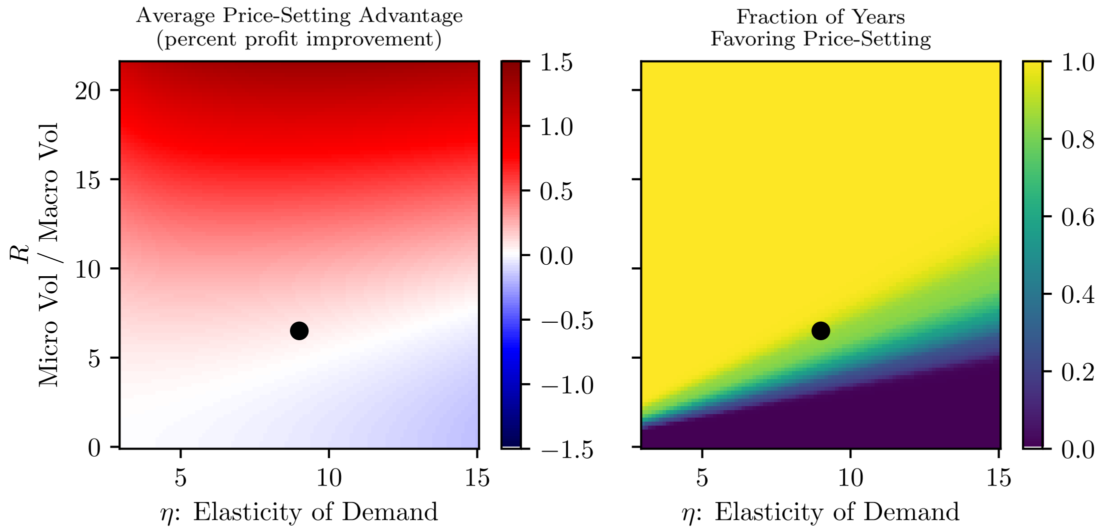
where it easily verified the above fraction is negative whenever $\eta\gamma < 1$ and can take on any value strictly less than zero. Moreover, when $\eta\gamma > 1$, $\Delta^P - \Delta^Q \geq 0$ if $\chi_1^P \geq \chi_1^Q$. This is true if and only if

$$\alpha_A \leq \frac{\kappa^A(\eta\gamma-1)}{1-\kappa^A(1-\eta\gamma)} \tag{183}$$

where it is easily verified that the above fraction is between zero and one if $\eta\gamma > 1$. Finally, the limiting result follows by noting that $\Delta^P - \Delta^Q$ is quadratic in α_A with a positive coefficient on the quadratic term. □

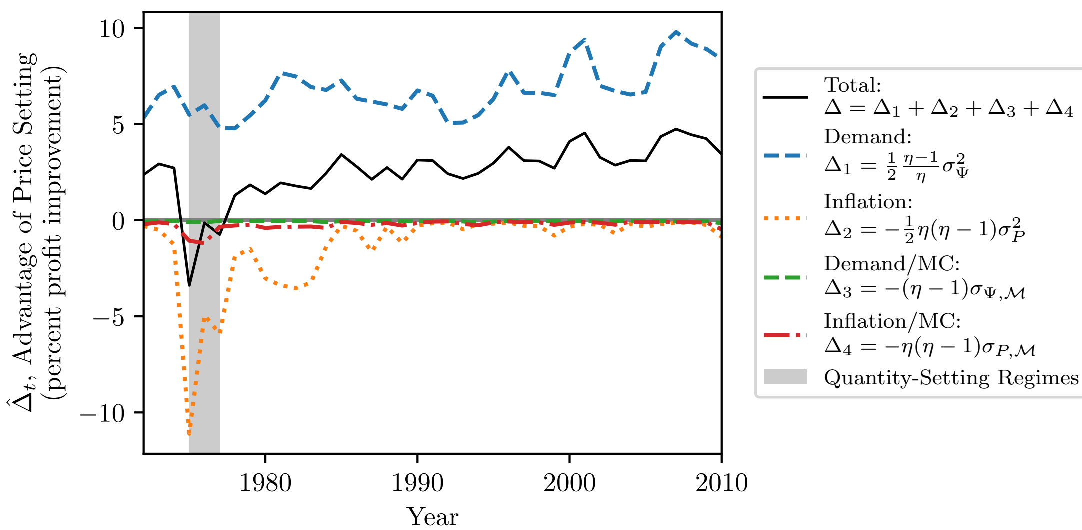
B Supplemental Tables and Figures

Figure 6: The Relative Benefit of Price-Setting in an Alternative, Annual Calculation



Note: This figure summarizes the relative advantage of price-setting for alternative values of the elasticity of demand (horizontal axis) and the ratio of microeconomic to macroeconomic volatility (vertical axis). The left panel plots the average advantage of price-setting over the sample, in units of 100 times log points (percent). The right panel plots the fraction of the sample in which price setting is optimal, or in which $\hat{\Delta}_t > 0$. In both panels, our baseline calibration is indicated with a solid dot.

Figure 7: The Relative Benefit of Price-Setting in an Alternative, Annual Calculation



Note: This figure plots our empirical estimate of $\hat{\Delta}_t$ (the comparative advantage of price-setting relative to quantity-setting) and its components, as defined in Proposition 1 (Equation 16), under a variant method with annual-frequency data and direct measurement of micro volatility from Bloom et al. (2018). Note that the time period (1972-2010) and time frequency (annual) differs from that in Figures 3 and 4 (quarterly, 1960 Q1 to 2022 Q4). The black line plots $\hat{\Delta}_t$, in units of expected percent profit improvement (100 times log points). The blue (dashed), orange (dotted), green (dashed), and red (dash-dotted) lines plot each of the four components of $\hat{\Delta}_t$, corresponding to uncertainty about different variables. The grey shading denotes periods in which $\hat{\Delta}_t < 0$ and thus, according to Proposition 1, quantity-setting is optimal for firms. As described in Section 6.1, the calculation uses estimates of time-varying volatilities from an annual-frequency CCC GARCH(1,1) model and a calibrated elasticity of demand $\eta = 9$

C Additional Theoretical Results

C.1 Decreasing Returns-To-Scale and Labor Disutility

This section characterizes the economy under general, iso-elastic decreasing returns to scale technology, and general Frisch elasticities for the labor supply.

Preferences are now given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \ln \frac{M_t}{P_t} - \int_0^1 \phi_{it} \frac{N_{kt}^{1+\psi}}{1+\psi} di \right) \right] \tag{184}$$

and production features decreasing returns-to-scale:

$$x_{it} = z_{it} A_t L_{it}^{\alpha} \tag{185}$$

so the benchmark case is nested when $\alpha = 1$ and $\psi = 0$. We solve the equilibrium fixed point problem under this more general specification. To this end, note that the wage schedule facing a firm is

$$w_{it} = \phi_{it} P_t C_t^{\gamma} L_{it}^{\psi} \tag{186}$$

we may also relate quantities to labour-hiring as follows

$$\left(\frac{x_{it}}{z_{it} A_t} \right)^{\frac{1}{\alpha}} = L_{it} \tag{187}$$

Quantity Setting. We first derive the dynamics of the economy under a quantity setting regime. To this end, a firm that sets quantities faces the following problem:

$$\max_{q_{it}} \mathbb{E}_{it} \left[\frac{C_t^{-\gamma}}{P_t} \left(\left(\frac{q_{it}}{\vartheta_{it} C_t} \right)^{-1/\eta} P_t q_{it} - P_t C_t^{\gamma} \phi_{it} \left(\frac{q_{it}}{z_{it} A_t} \right)^{\frac{\psi+1}{\alpha}} \right) \right] \tag{188}$$

The optimal quantity set by firms is therefore given by:

$$\log q_{it} = -\frac{\alpha\eta}{\alpha + \psi\eta + \eta(1-\alpha)} \left[\log \left(\frac{\eta(\psi+1)}{\alpha(\eta-1)} \right) + \left(\mathbb{E} \left[\phi_{it} \left(z_{it} A_t \right)^{-\frac{\psi+1}{\alpha}} \right] \right) - \left(\mathbb{E} \left[\vartheta_{it}^{\frac{1}{\eta}} C^{-\gamma + \frac{1}{\eta}} \right] \right) \right] \tag{189}$$

Our solution strategy is identical to the one in the main text: we conjecture a log-linear solution for aggregate consumption (42), which we use to obtain a log-linear expression for q_{it} in terms of aggregates. We then substitute this expression into the consumption index (18) and solve the fixed point. We may then obtain the following characterization of aggregate consumption and the price level in a quantity setting regime.

Proposition 10. *If all firms set quantities, output in the unique log-linear temporary equilibrium is given by:*

$$\log C_t = \chi^Q_{0,t-1} + \frac{\eta(\psi+1)\kappa^A_{t-1}}{\eta(1+\psi-\alpha)+\alpha\left(1-\kappa^A_{t-1}(1-\eta\gamma)\right)}\log A_t \tag{190}$$

where and $\chi^Q_{0,t-1}$ *is a constant that depends only on parameters and past shocks to the economy. The aggregate price level follows:*

$$\log P_t = \tilde{\chi}^Q_{0,t-1} + \frac{\gamma\eta(\psi+1)\kappa^A_{t-1}}{\eta(1+\psi-\alpha)+\alpha\left(1-\kappa^A_{t-1}(1-\eta\gamma)\right)}\log A_t + \log M_{t+1} \tag{191}$$

where $\chi^Q_{0,t-1}$ *and* $\tilde{\chi}^Q_{0,t-1}$ *are constants that depend only on parameters and past shocks to the economy.*

Note that setting $\psi = 0$ and $\alpha = 1$ yields the result in the main text. We note how the presence of more general forms of labor disutility and decreasing returns to scale change the responsiveness of consumption to output. First, it is straightforward to see that $\chi^Q_{A,t-1}$ is globally increasing in ψ for all parameter values, and that

$$\lim_{\psi\to\infty}\log C_t = \kappa^A_{t-1} \tag{192}$$

Hence, the presence of ψ can *increase* the response of consumption to productivity shocks (relative to the baseline case) if and only if $\gamma > 1$. This is because large values for ψ effectively eliminate wealth effects on the choice of labour, thereby making $\log C_t$ independent of γ.

The effect of α on $\chi^Q_{A,t-1}$ is more nuanced. If κ^A_{t-1} is sufficiently low, increasing α raises the responsiveness of consumption to productivity shocks. This captures the standard effect of reducing the concavity inherent in the production function. If κ^A_{t-1} is large and $\gamma > 1$, $\chi^Q_{A,t-1}$ decreases. For large values of γ and signal-to-noise ratios, firms respond to positive signals about productivity by increasing their demand for labor, on average. This increased demand pushes up wages through wealth effects, which has a counteracting force on firm demand in general-equilibrium. This "negative" demand component increases faster than the direct, partial-equilibrium effect with respect to α when labour is sufficiently responsive to wages (i.e. $\gamma > 1$).

Price Setting. We now turn our attention to the firm's problem under a price-setting regime. Under price-setting, the firms solve:

$$\max_{p_{it}} \mathbb{E}_{it}\left[p_{it}\frac{C_t^{-\gamma}}{P}\left(\left(\frac{p_{it}}{P_t}\right)^{-\eta}\vartheta_{it}C_t - \phi_{it}P_tC_t^{\gamma}\left(\frac{p_{it}}{P_t}\right)^{-\frac{\eta(1+\psi)}{\alpha}}\left(\frac{z_{it}A_t}{\vartheta_{it}C_t}\right)^{\frac{1+\psi}{\alpha}}\right)\right] \tag{193}$$

The optimal price set by firms is therefore given by:

$$\log p_{it} = \frac{\alpha}{\alpha+\eta(1+\psi-\alpha)}\left[\log\frac{\eta(\psi+1)}{\alpha(\eta-1)} + \log\mathbb{E}_{it}\left[\phi_{it}\left((z_{it}A_t)^{-1}P_t^{\eta}\vartheta_{it}C_t\right)^{\frac{1+\psi}{\alpha}}\right] - \log\mathbb{E}_{it}[C_t^{1-\gamma}P_t^{\eta-1}\vartheta_{it}]\right] \tag{194}$$

We may solve for the fixed point as above. This yields the following proposition.

Proposition 11. *The aggregate price level in the unique log-linear equilibrium is given by*

$$\log P_t = \log\chi^P_{0,t-1} + \chi^P_{A,t-1}\log A_t + \chi^P_{M,t-1}\log M_t \tag{195}$$

where

$$\chi^P_{A,t-1} = \frac{-(1+\psi)\kappa^A_{t-1}}{(\alpha+\eta(1+\psi-\alpha)) - \left(\eta-\frac{1}{\gamma}\right)(1+\psi-\alpha)\kappa^A_{t-1}} \tag{196}$$

$$\chi^P_{M,t-1} = \frac{\frac{1}{\gamma}(1+\psi-\alpha(1-\gamma))\kappa^M}{(\alpha+\eta(1+\psi-\alpha)) - \left(\eta-\frac{1}{\gamma}\right)(1+\psi-\alpha)\kappa^M} \tag{197}$$

and aggregate consumption follows

$$\log C_t = \tilde{\chi}^P_{0,t-1} - \frac{1}{\gamma}\chi^P_{A,t-1}\log A_t + \frac{1}{\gamma}(1-\chi^P_{M,t-1})\log M_t \tag{198}$$

Note again that letting $\psi = 0$ and $\alpha = 1$ yields the corresponding proposition in the main text. An interesting observation is that, under price setting, increasing ψ makes the price level more responsive to money. High values of ψ increase the responsiveness of marginal costs to aggregate demand, thereby inducing firms to increase their prices further in response to perceived changes in aggregate demand conditions.

C.2 Adjustment Costs

Suppose that the firm is subject to adjustment costs in prices and quantities of the form $\delta_P \mathbb{V}[\log p]$ and $\delta_Q \mathbb{V}[\log q]$. For any fixed q, price adjustment costs are:

$$C^Q = \delta_P \mathbb{V}[-\frac{1}{\eta}\log q + \log P + \frac{1}{\eta}\log \Psi] = \delta_P(\sigma_P^2 + \frac{1}{\eta^2}\sigma_\Psi^2 + \frac{2}{\eta}\sigma_{P,\Psi}) \tag{199}$$

For any fixed p, quantity adjustment costs are:

$$C^P = \delta_Q \mathbb{V}[-\eta \log p + \log \Psi + \eta \log P] = \delta_Q(\sigma_\Psi^2 + \eta^2\sigma_P^2 + 2\eta\sigma_{\Psi,P}) \tag{200}$$

Thus:

$$C^Q - C^P = (1 - \frac{\delta_Q}{\delta_P}\eta^2)C^Q = (1 - \frac{\delta_Q}{\delta_P}\eta^2)\delta_P(\sigma_P^2 + \frac{1}{\eta^2}\sigma_\Psi^2 + \frac{2}{\eta}\sigma_{P,\Psi}) \tag{201}$$

or:

$$C^Q - C^P = \left(\frac{1}{\eta}\delta_P - \eta\delta_Q\right)(\frac{1}{\eta}\sigma_\Psi^2 + \eta\sigma_P^2 + 2\sigma_{P,\Psi}) \tag{202}$$

Prices are preferred to quantities if and only if $V^P - V^Q \geq C^P - C^Q$. When $\delta_P = \eta^2\delta_Q$, this reduces to main analysis. Otherwise, we have that $V^P - V^Q = (\exp\{\Delta\} - 1)V^Q$. So, we can write the condition as:

$$(\exp\{\Delta\} - 1)\, V^Q \geq \left(\eta\delta_Q - \frac{1}{\eta}\delta_P\right)(\frac{1}{\eta}\sigma_\Psi^2 + \eta\sigma_P^2 + 2\sigma_{P,\Psi}) \tag{203}$$

We also know that:

$$V^Q = \frac{1}{\eta - 1}\left(\frac{\eta}{\eta - 1}\right)^{-\eta} \mathbb{E}\left[\Lambda\mathcal{M}\right]^{1-\eta} \mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}]^{\eta} \tag{204}$$

$$\log \mathbb{E}[\Lambda\mathcal{M}] = \log \mathbb{E}[\exp\{\log\Lambda + \log\mathcal{M}\}] = \mu_\Lambda + \mu_\mathcal{M} + \frac{1}{2}\left(\sigma_\Lambda^2 + \sigma_\mathcal{M}^2\right) + \sigma_{\Lambda,\mathcal{M}} \tag{205}$$

$$\log \mathbb{E}\left[\Lambda\Psi^{\frac{1}{\eta}}\right] = \log \mathbb{E}\left[\exp\left\{\log\Lambda + \frac{1}{\eta}\log\Psi\right\}\right] = \mu_\Lambda + \frac{1}{\eta}\mu_\Psi + \frac{1}{2}\left(\sigma_\Lambda^2 + \frac{1}{\eta^2}\sigma_\Psi^2\right) + \frac{1}{\eta}\sigma_{\Lambda,\Psi} \tag{206}$$

Thus:

$$V^Q = \frac{1}{\eta - 1}\left(\frac{\eta}{\eta - 1}\right)^{-\eta} \exp\left\{\mu_\Lambda + (1-\eta)\mu_\mathcal{M} + \mu_\Phi + \frac{1}{2}\left(\sigma_\Lambda^2 + (1-\eta)\sigma_\mathcal{M}^2 + \frac{1}{\eta}\sigma_\Psi^2 + (1-\eta)\sigma_{\Lambda,\mathcal{M}} + \sigma_{\Lambda,\Psi}\right)\right\} \tag{207}$$

So we require that:

$$\exp\{\Delta\}-1 \geq \frac{\left(\eta\delta_Q - \frac{1}{\eta}\delta_P\right)\left(\frac{1}{\eta}\sigma_\Psi^2 + \eta\sigma_P^2 + 2\sigma_{P,\Psi}\right)}{\frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\exp\Big\{\mu_\Lambda + (1-\eta)\mu_{\mathcal{M}} + \mu_\Phi + \frac{1}{2}\left(\sigma_\Lambda^2 + (1-\eta)\sigma_{\mathcal{M}}^2 + \frac{1}{\eta}\sigma_\Psi^2 + (1-\eta)\sigma_{\Lambda,\mathcal{M}} + \sigma_{\Lambda,\Psi}\right)} \tag{208}$$

When Δ is small, we also know the LHS is approximately Δ. So, approximately:

$$\begin{aligned}&\frac{1}{2}(\eta-1)\left(\frac{1}{\eta}\sigma_\Psi^2 - \eta\sigma_P^2 - 2\sigma_{\Psi,\mathcal{M}} - 2\eta\sigma_{P,\mathcal{M}}\right)\\ &\geq \frac{\left(\eta\delta_Q - \frac{1}{\eta}\delta_P\right)\left(\frac{1}{\eta}\sigma_\Psi^2 + \eta\sigma_P^2 + 2\sigma_{P,\Psi}\right)}{\frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\exp\left\{\mu_\Lambda + (1-\eta)\mu_{\mathcal{M}} + \mu_\Phi + \frac{1}{2}\left(\sigma_\Lambda^2 + (1-\eta)\sigma_{\mathcal{M}}^2 + \frac{1}{\eta}\sigma_\Psi^2 + (1-\eta)\sigma_{\Lambda,\mathcal{M}} + \sigma_{\Lambda,\Psi}\right)\right\}}\end{aligned} \tag{209}$$

C.3 Mixed Equilibria

We may also consider "mixed" regimes, in which a fraction $\lambda_t \in (0,1)$ of firms set prices at time t. We first expand our definition of a temporary equilibrium to allow for mixing.

Definition 3 (Temporary Equilibrium with Mixing)**.** *A temporary equilibrium is a partition of $\mathbb{N}$ into three sets $\mathcal{T}^P$, $\mathcal{T}^Q$, and $\mathcal{T}^{PQ}$ and a collection of variables*

$$\left\{\{p_{it}, q_{it}, C_{it}, N_{it}, L_{it}, w_{it}, \phi_{it}, \vartheta_{it}, z_{it}, \Pi_{it}\}_{i\in[0,1]}, C_t, P_t, M_t, A_t, B_t, N_t, \Lambda_t, \lambda_t\right\}_{t\in\mathbb{N}} \tag{210}$$

such that:

1. *In periods $t \in \mathcal{T}^P$, all firms choose their prices p_{it} to maximize expected real profits under the household's real stochastic discount factor.*
2. *In periods $t \in \mathcal{T}^Q$, all firms choose their quantities q_{it} to maximize expected real profits under the household's real stochastic discount factor.*
3. *In periods $t \in \mathcal{T}^{PQ}$ a fraction λ_t of firms choose prices p_{it} and a fraction $(1-\lambda_t)$ choose quantities q_{it} to maximize expected real profits under the households' real stochastic discount factor.*
4. *In all periods, the household chooses consumption C_{it}, labor supply N_{it}, money holdings M_t, and bond holdings B_t to maximize their expected utility subject to their lifetime budget constraint, while Λ_t is the household's marginal utility of consumption.*
5. *In all periods, money supply M_t and productivity A_t and evolve exogenously via Equations 21 and 23.*

6. *In all periods, firms' and consumers' expectations are consistent with the equilibrium law of motion.*
7. *In all periods, the markets for the intermediate goods, final good, labor varieties, bonds, and money balances all clear.*

As in the main text, we define an *equilibrium* as a temporary equilibrium in which the choice of setting prices or quantities is optimal. In the case where there is mixing, firms are indifferent between price or quantity-setting.

Definition 4 (Equilibrium with Mixing)**.** *An equilibrium is a temporary equilibrium in which:*

1. *If $t \in \mathcal{T}^P$, all firms find price-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*
2. *If $t \in \mathcal{T}^Q$, all firms find quantity-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*
3. *If $t \in \mathcal{T}^{PQ}$ firms are indifferent between price or quantity-setting.*

The presence of mixing complicates the equilibrium characterization because it generally leads to a solution for aggregate consumption that is not log-linear in aggregates, thereby making it difficult to characterize in closed form. We address this challenge by taking a log-linear approximation of the aggregate price level (19) around a zero innovation limit at $t-1$, which we denote by P_{t-1}^{full}. Approximating (19) in this way yields the following, to first-order in the shocks A_t and M_t:

$$\log P_t = P_{t-1}^{full} + \lambda_t \mathbb{E}\left[\log p_{it} | \text{price setting}\right] + (1-\lambda_t)\mathbb{E}\left[\log p_{it} | \text{quantity setting}\right] \tag{211}$$

where the expectations are over the cross-sectional distribution of firms, conditional on their choice to set prices or quantities. Crucially, we still allow firms best responses to be fully non-linear. This feature implies that our approximate formulas for the dynamics of the economy under mixing will *exactly* equal our fully non-linear characterization in the main text whenever $\lambda_t = 0$ or $\lambda_t = 1$. The following proposition characterizes the dynamics of the economy in the unique log-linear approximate equilibrium for $\lambda_t \in [0,1]$. We present the proposition under the conditions that permit active monetary policy (outlined in Section 5), and only note that the special case of $\alpha_A = 0$ can also encompass time-varying volatility in output.

Proposition 12. *Equilibrium prices and consumption in a mixed regime are given by the following expressions, to first-order:*

$$\log P_t = \chi^{PQ}_{0,t-1} + \chi^{PQ}_{A,t-1}(\lambda_t) \log A_t + \chi^{PQ}_{A,t-1}(\lambda_t) \log M_t \tag{212}$$

$$\log C_t = \tilde{\chi}^{PQ}_{0,t-1} - \frac{1}{\gamma}\chi^{PQ}_{A,t-1}(\lambda_t) \log A_t + \frac{1}{\gamma}\left(1 - \chi^{PQ}_{M,t-1}(\lambda_t)\right) \log M_t \tag{213}$$

where

$$\chi^{PQ}_{A}(\lambda_t) = \frac{\eta\gamma\kappa^A\left(\alpha_A - 1\right) - (1-\lambda_t)\alpha_A\left(\kappa^A - 1\right)}{\eta\gamma - (1-\lambda_t)\left(\eta\gamma - 1\right)\left(1 - \kappa^A\right)} \tag{214}$$

$$\chi^{PQ}_{m}(\lambda_t) = \frac{\eta\gamma\lambda_t\kappa^m + (1-\lambda_t)\left(\kappa^m\left(\eta\gamma - 1\right) + 1\right)}{\eta\gamma\lambda_t + (1-\lambda_t)\left(\kappa^m\left(\eta\gamma - 1\right) + 1\right)}\sigma_m \tag{215}$$

Proof. See Appendix D.1. □

It is straightforward to verify that setting $\lambda_t = 0$ or $\lambda_t = 1$ gives us the dynamics for prices and consumption for "pure" equilibria considered in Section 5.

D Proofs of Additional Theoretical Results

D.1 Proof of Proposition 12

Proof. We guess that the price level is a log-linear function of the money supply shock and productivity:

$$\log P_t = \log \chi_0 + \chi_1 \log A_t + \chi_2 \log m_t \tag{216}$$

Note also that equation (35) implies that the dynamics for consumption are given by

$$\log C_t = -\frac{1}{\gamma}\left(\log(\chi_0) - \log\left(\frac{i^*}{1+i^*}\right)\right) - \frac{1}{\gamma}(\chi_1 - \alpha_A)\log A_t - \frac{1}{\gamma}(\chi_2 - \sigma_m)\log m_t \tag{217}$$

We now consider the first expectation term in Equation (211), which the cross-sectional average of log-prices for all *price-setters*. Following through Equations (153)-(158) in A.9, we can collect all terms that depend on $\log A_t$ and $\log m_t$ to obtain:

$$\log A_t: \quad (\alpha_A - 1)\kappa_A \tag{218}$$

$$\log m_t: \quad \sigma_m \kappa_m \tag{219}$$

We now consider the second expectation term in Equation (211), which is the cross-sectional average of log-prices for all *quantity-setters*. Using (43), this is given by

$$\mathbb{E}\left[\log\left(\frac{\eta}{\eta-1}\right) + \log \mathbb{E}_{it}\left[\phi_{it}(z_{it}A_t)^{-1}\right] - \mathbb{E}_{it}\left[\vartheta_{it}^{\frac{1}{\eta}} C_t^{-\gamma+\frac{1}{\eta}}\right] + \frac{1}{\eta}\log C_t + \log P_t\right] \tag{220}$$

Simplifying this expression and collecting terms for $\log A_t$ and $\log m_t$ separately yields

$$\log A_t: \quad -\left(1 + \chi_1\left(1 - \frac{1}{\eta\gamma}\right)\right)\kappa_A + \chi_1\left(1 - \frac{1}{\eta\gamma}\right) + \alpha_A\kappa_A\left(1 - \frac{1}{\eta\gamma}\right) \tag{221}$$

$$\log m_t: \quad \chi_2\left(1 - \frac{1}{\eta\gamma}\right)(1-\kappa_m) + \left(1 - \frac{1}{\eta\gamma}\right)\sigma_m\kappa_m + \frac{1}{\eta\gamma}\sigma_m \tag{222}$$

We may now equate coefficients using (211). This yields an equation for χ_1:

$$\chi_1 = \lambda_t(\alpha_A - 1)\kappa_A + (1-\lambda_t)\left[-\left(1 + \chi_1\left(1 - \frac{1}{\eta\gamma}\right)\right)\kappa_A + \chi_1\left(1 - \frac{1}{\eta\gamma}\right) + \alpha_A\kappa_A\left(1 - \frac{1}{\eta\gamma}\right) + \frac{1}{\eta\gamma}\alpha_A\right] \tag{223}$$

We can similarly obtain equation for χ_2:

$$\chi_2 = \lambda_t\sigma_m\kappa_m + (1-\lambda_t)\left(\frac{1}{\eta\gamma} + \kappa_m\left(\frac{1}{\eta\gamma} - 1\right)\right)\sigma_m\kappa_m \tag{224}$$

Solving these two equations yields:

$$\chi_1(\lambda_t) = \frac{\eta\gamma\kappa_A\left(\alpha_A - 1\right) - (1-\lambda_t)\alpha_A\left(\kappa_A - 1\right)}{\eta\gamma - (1-\lambda_t)\left(\eta\gamma - 1\right)\left(1 - \kappa^A_{t-1}\right)} \tag{225}$$

$$\chi_2(\lambda_t) = \frac{\eta\gamma\lambda_t\kappa_m + (1-\lambda_t)\left(\kappa_m\left(\eta\gamma - 1\right) + 1\right)}{\eta\gamma\lambda_t + (1-\lambda_t)\left(\kappa_m\left(\eta\gamma - 1\right) + 1\right)}\sigma_m \tag{226}$$

The proof follows. □